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Intuitionistic Fuzzy Multi-objective Optimization Method for Determination of Optimal Cropping Pattern

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Abstract

Agriculture has become a difficult occupation due to inadequate farming resources and cultivation risks. Thus, proficient utilization of resources alongside risk-alleviation strategies is essential aspect to realize sustainable farm benefits. Most of the earlier studies have reported the capability of Operations research in solving agricultural problems and enhancing farm productivity. However, they have not addressed effectively the distinctive nature of decision-makers, uncertainties, and associated risks of agriculture. This study mainly aims to fill these lacunae by applying an intuitionistic fuzzy optimization method to determine optimal cropping pattern that maximizes overall net benefits, minimizing cultivation costs and workforce concurrently with regard to procurable agricultural resources. For this purpose, an effective multi-objective optimization method is formulated, and its effectiveness is verified through proof and numerical example. The comparison between existing and proposed cropping patterns showed that the proposed patterns offer several advantages in enhancing overall agricultural benefits sustainably for farmers in the study area.

Keywords: Multi-objective optimization; Intuitionistic fuzzy optimization; Agricultural production planning; Cropping pattern.

1 Introduction

Today, the burgeoning world's population increases the demand for agricultural products and this in turn increasing pressure on the resources required for production (Wang, 2022). However, to meet the ever-escalating demand for nourishment, crop production must be boosted either by increasing land area for cultivation or by enhancing production per unit area of land (FAO, 2017; Mirkarimi et al., 2013). Since agricultural resources for farming are very limited all over the world (Guo et al., 2021), increasing land area for crop production regardless of limiting factors causes deterioration of available resources. Moreover, climate change, drought, political disputes, and disease are holding back agricultural advancement and remain key determinants of food security and poverty alleviation (Luo et al., 2023; Zerssa et al., 2021). Therefore, it is crucial to design means of efficient utilization of resources and risk alleviation strategies to improve the overall agricultural returns sustainably.

In this regard, the decision of agricultural stakeholders has its own impact on the achievement of the desired objectives. Even if decision-making for the optimum utilization of resources is a challenging task for farmers and agricultural managers, it can be scientifically addressed using optimization models to support farm activities and contribute to feeding the steadily growing population (Carravilla & Oliveira, 2013; Weintraub & Romero, 2006).

Agriculture is the primary sector of the Ethiopian economy, employing approximately 85% of the country's population as workers (Zerssa et al., 2021). It contributes 50% of Ethiopia's gross domestic product and earns over 90% of the foreign exchange (Haile & Kasa, 2015; Zerssa et al., 2021). The country's goal for achieving overall economic growth mainly depends on the accomplishment of agriculture sector (Haile & Kasa, 2015).

Despite the country's vast irrigable land and water resources, farming is weather dependent and the production of which depends heavily on the availability of rainfall (Awulachew & Ayana, 2011). Consequently, most farmers are exposed to inconsistent rainfall patterns and weather conditions (Zerssa et al., 2021). Even though Ethiopia is vulnerable to the vagaries of natural weather conditions, it has substantial agricultural potential because of its vast areas of fertile land, enormous labor, abundant water resources, and diverse climate conditions (Awulachew & Ayana, 2011; Kelbore, 2014).

Due to seasonal variation, Ethiopia has three distinct seasons: Kiremt

(June – September), Bega (October – January), and Belg (February – May) (Gebremichael et al., 2014). Kiremt is the main rainy season, accounting for about 90% of agricultural production, while Bega is dry and Belg is a short rainy season contributing the remaining 10% of production (Kelbore, 2014).

A cropping pattern is the proportion of land area under various crops that changes over space and time (G. Singh, 2012). It is the annual succession of different crops and fallow in a particular region and can be reported at the farm level to address the collective issues of a farming system (Andrews & Kassam, 1976).

The principal objective of optimal cropping pattern (OCP) is to identify the combination of several crops to be cultivated which maximizes the net return of farming by managing agricultural risks with respect to available resources (Ouda et al., 2017).

As the population grows, agricultural resources have been decreasing and the situation worsens with the spread of drought (Carravilla & Oliveira, 2013). These circumstances continue to generate many agricultural inquiries in search for more productive alternatives on a given land area for the optimal utilization of other agricultural resources (Paudel, 2016). Therefore, OCP is one of the important and feasible mechanisms to increase productivity with other integrated scientific agricultural practices (Luo et al., 2023; Ouda et al., 2017).

Determining the OCP remains a complex task for farmers and managers (Pawar et al., 2026), as decisions on machinery selection, input use, operation timing, and cultivation practices must be made in each cropping season (Duan et al., 2021).

In Ethiopia, where most farmers practice traditional agriculture, these decisions are largely based on experience, fluctuating market prices, and other operational factors (Kelbore, 2014). Conversely, policies and guidelines delivered through extension services often fail to account for multiple agricultural objectives and constraints, such as weather variability and production risk. Therefore, more effective strategies are required through the use of optimization models.

The problem of optimizing multiple goals simultaneously under given constraints is called multi-objective optimization (MOO). It involves concurrent optimization of incommensurate and conflicting goals subject to different constraints (V. Singh & Yadav, 2018).

Many real-life problems are inherently characterized by multiple and conflicting goals with uncertainties (Gupta et al., 2000). So, it is difficult to deal with such problems using classical optimization techniques. Hence, intuitionistic fuzzy sets (IFSs) can be used to represent insufficient information, imprecise concept, uncertainties and the diverse perspectives of decision-makers (DMs) in a more generalized way compared to fuzzy sets (Roszkowska et al., 2024). Thus, intuitionistic fuzzy modelling is more relevant than other classical optimization methods (Pawar et al., 2026).

Several researchers have attempted to address agricultural production planning (APP) problems employing various MOO methods to recommend an alternative cropping system for improved outcomes (Luo et al., 2023; Weintraub & Romero, 2006). However, the ambiguity of the parameters in the problem, inconsistent natural conditions, the distinctive perspective of DMs, and associated operational risks in agriculture have not been well addressed in their studies.

To overcome the above difficulties, it is imperative to formulate an effective optimization model and design OCP at the farm level by considering the available resources to assist sustainable crop production.
This study aims to investigate the application of an intuitionistic fuzzy multi-objective optimization (IFMOO) model and to propose OCP for the study area that improves farmers' welfare concerning scarce resources and reduces various challenges of farming.

The rest of this paper is organized as follows. In Section 2, the related literature is reviewed. The optimization method for addressing the practical problem is outlined in Section 3. A comparative analysis of the proposed method is presented in Section 4. Section 5 provides the model application in three subsections: study area description, data collection and analysis, and problem formulation. A detailed analysis of the results is given in Section 6. Section 7 summarizes the findings, discusses limitations, and provides recommendations for future research.

2 Literature Review

Agriculture is one of the fields where Operations research (OR) models were first employed and they have been extensively applied (Rădulescu et al., 2014). The capability of addressing MOO problems for decision-making helps out OR to play a fundamental role in agriculture (Rădulescu et al., 2014; Weintraub & Romero, 2006).

Carravilla and Oliveira (2013) reviewed studies demonstrating the applications of OR in agriculture at farm and sector levels. Depending on problem complexity, some used linear programming (LP), while others employed multi-objective and fuzzy optimization models to address APP problems.

Weintraub and Romero (2006) demonstrated the potential of OR in the management of agricultural resources and forestry, and their advantages in simplifying DM in farming activities. In their article, the applications of OR in APP problems at the farm and regional levels were comprehensively reviewed. They address uncertainties, risks, environmental conservation, and discuss future research directions in these areas.

Environmental, social, and economic factors make agricultural data inherently uncertain (Bairwa et al., 2013), leading to decision-making under ill-defined objectives and constraints (Li et al., 2017). Accordingly, risks in agriculture are better represented with fuzzy numbers than crisp values, prompting the development and application of fuzzy multi-objective optimization (FMOO) methods to handle APP problems.

Although many researchers employed FMOO approaches to deal with APP problems (Amini, 2015; Biswas & Pal, 2005; Gupta et al., 2000; Mirajkar & Patel, 2012; Mirkarimi et al., 2013; Rasikh et al., 2024; Wang, 2022; X. Zeng et al., 2010) their studies were limited in scope and did not represent the real nature of the problems very well (Mahapatra & Roy, 2009). This is due to insufficient information, incomplete attributes, ill-definedness, uncertainties, and vagueness in every aspect of MOO problems (Sen et al., 2018). Consequently, different advanced generalizations are ascertained. From that, IFS, which is originated by Atanassov (1986), is an effective generalization of fuzzy sets. It has been used in a wide range of operations because of its ability to address uncertainties and vagueness in practical problems. Thus, intuitionistic fuzzy optimization (IFO) (Angelov, 1995) was introduced to handle different pragmatic problems.

IFO technique is a relatively recent research field in contrast to fuzzy optimization approaches (Angelov, 1995). It enhances understanding of the addressed problems and provides valuable insights into their nature (Roszkowska et al., 2024). Moreover, the output of an investigation employing IFO is a more valuable analytical means for researchers, practitioners, and experts.

In APP problem, interactions among various natural entities and correlated factors complicate the management process. These factors enforce DMs to use advanced optimization models for proficient usage of resources and to gain better overall benefits.

Nishad and Singh (2015) employed an intuitionistic fuzzy goal programming to resolve the land use planning problem. They considered the agricultural system undertaken by Biswas and Pal, 2005 in which several seasonal crops were cultivated in a year and different productive resources are taken into account in the model. Their study revealed that IFO method gives better results in all aspects compared to the results obtained by the fuzzy optimization method.

Li et al. (2017) formulated an IFMOO model that incorporates MOO, nonlinear programming (NLP) and intuitionistic fuzzy number (IFN) to deal with the uncertainties of conflicting targets in irrigated agriculture to support sustainable farming. They employed an IFMOO model to allocate limited accessible water to rice during growth stages to maximize crop yield, minimize utilized water and cost of water supply in Heping irrigation area of northeast China.

Li et al. (2019) used a multi-objective NLP model in an intuitionistic fuzzy environment (IFE) for the management of the water-energy-food nexus in irrigational agriculture, considering the cropping system of the Heihe River basin in northwest China. In their study, both optimistic and pessimistic views of DMs were considered under different scenarios to maximize system profit and minimize carbon dioxide emissions, subject to water, energy, land, and other input resources for cultivating wheat, corn, and vegetables in the three regions of the basin.

Pawar et al., 2022 applied an IFMOO method to determine the OCP of the Ukai irrigation area in India. Their approach combined minimizing the aggregated hesitation level of objectives with maximizing the minimum membership degree and minimizing the maximum non-membership degree. This produced an OCP that maximized net returns and employment, while minimizing farming expenditures under constraints of arable land and irrigation water.

Li et al., 2020 developed an optimization model for sustainable irrigated agriculture combining IFMOO, nonlinear mixed-integer, and fuzzy credibility-constrained programming. Their study aimed to allocate water and farmland to crops across subareas and seasons to optimize net returns while considering socioeconomic and ecological objectives. The algorithm was successfully applied to crop planning in the Heping irrigation area of China.

Kousar et al., 2022 formulated an IFO method considering all parameters and variables as IFN to optimize the production of five types of fruits in Baluchistan region, Pakistan.Their results showed that the fully IFO technique has an imperative advantage to consider the fluctuating nature of prices and input resources more efficiently.

L. Zeng et al. (2020) studied sustainable resource management for the Zhanghe Reservoir irrigation in central China using an interval stochastic multi-objective mixed-integer model in an IFE. Their objectives were to maximize crop production, hydroelectricity, and water allocation while optimizing cropland under constraints of water availability, crop demand, land policy, and electricity generation. The results demonstrated efficient farmland and water management to support food security and mitigate global warming sustainably.

A few researchers have used IFMOO models to handle uncertainties and risks in APP problems. As noted above, investigators have applied various IFMOO techniques to suggest alternative cropping patterns for better outcomes. However, they have not effectively addressed differences among DMs, uncertainties, and risks that severely affect crop production.
For instance, in most studies, parameters are not consistently considered as IFNs, constraints are not incorporated into the solution framework, DM preferences and stakeholder interactions are overlooked, and the sustainability of objectives is inadequately addressed.

To overcome these shortcomings, the present study formulates an IFMOO model to design an OCP that maximizes sustainable net benefits for a large-scale farm (LSF) and farmers in Gefersa kebele. It also aids farmers, development agents, and extension workers in complex decision-making under multiple perspectives and scenarios for a LSF in Abeshge district.

3 IFO Method

3.1 Preliminaries

Definition 3.1.1 (Atanassov, 1986). An IFS $\tilde{A}$ in the universe X is given by $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle : x \in X\}$, where $\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x), \pi_{\tilde{A}}(x):$ $X \to [0,1]$ such that
$0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1$ and $\pi_{\tilde{A}}(x) = 1 - (\mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x))$, $\forall x \in X$. The values $\mu_{\tilde{A}}(x)$, $\nu_{\tilde{A}}(x)$ and $\pi_{\tilde{A}}(x)$ describe the degree of membership, non-membership, and indeterminacy of x being in $\tilde{A}$, respectively.
Definition 3.1.2 (Mahapatra & Roy, 2009). A triangular IFN $\tilde{A}$ is an IFS, represented as $\tilde{A} = \langle a_1, a_2, a_3; a_1', a_2, a_3'\rangle$, where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$, with

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{if } a_1 < x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & \text{if } a_2 \le x < a_3 \\ 0, & \text{otherwise,} \end{cases}$$

and

$$\nu_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, & \text{if } a_1' < x \le a_2 \\ \frac{x-a_2}{a_3'-a_2}, & \text{if } a_2 \le x < a_3' \\ 1, & \text{otherwise.} \end{cases}$$

Definition 3.1.3 (S. K. Singh & Yadav, 2015a).
Let $\tilde{A} = \langle a_1, a_2, a_3; a_1', a_2, a_3'\rangle$ be a triangular IFN, the accuracy function of $\tilde{A}$ is denoted by $\Gamma(\tilde{A})$ and defined as $\Gamma(\tilde{A}) = \frac{a_1'+a_1+4a_2+a_3+a_3'}{8}$.
Accuracy function Γ is used to defuzzify IFNs.

In handling of practical problems, vagueness and uncertainty can be addressed in an IFE by considering parameters as IFNs and treating inequality and equality as intuitionistic fuzzy inequalities and equality. Based on this principle, an IFMOO problem is formulated (V. Singh & Yadav, 2018)

$$\begin{aligned} & \max\,\{\widetilde{f_1}(X), \widetilde{f_2}(X), ..., \widetilde{f_{k_1}}(X)\}, \\ & \min\,\{\widetilde{f_{k_1+1}}(X), \widetilde{f_{k_1+2}}(X), ..., \widetilde{f_k}(X)\} \\ & \qquad \text{subject to} \\ & \widetilde{g_i}(X) \precsim \widetilde{c}_i,\ i = 1, 2, \ldots, m_1, \\ & \widetilde{g_i}(X) \succsim \widetilde{c}_i,\ i = m_1+1, m_1+2, \ldots, m_2, \\ & \widetilde{g_i}(X) \approx \widetilde{c}_i,\ i = m_2+1, m_2+2, \ldots, m, \\ & X \ge 0, \end{aligned} \tag{1}$$

where $\widetilde{f_j}(X)$ and $\widetilde{g_i}(X)$ are intuitionistic fuzzy functions, X is n-dimensional variable $\forall j = 1, 2, ..., k$ and $\forall i = 1, 2, ..., m$.

To reformulate problem (1) as an equivalent crisp MOO problem, each parameter has to be defuzzified applying the accuracy function (Rădulescu et al., 2014). Then, the degrees of acceptance and rejection of the objectives have to be described to form a single objective optimization problem. Accordingly, the solution of the IFMOO problem can be found by solving such an equivalent single objective problem (S. K. Singh & Yadav, 2015b).

The IFMOO problem (1) can be changed into the following equivalent deterministic MOO problem (V. Singh & Yadav, 2018):

$$\begin{aligned} & \max\,\{f_1(X), f_2(X), ..., f_{k_1}(X)\}, \\ & \min\,\{f_{k_1+1}(X), f_{k_1+2}(X), ..., f_k(X)\} \\ & \qquad \text{subject to} \\ & g_i(X) \le c_i,\ i = 1, 2, \ldots, m_1, \\ & g_i(X) \ge c_i,\ i = m_1+1, m_1+2, \ldots, m_2, \\ & g_i(X) = c_i,\ i = m_2+1, m_2+2, ..., m, \\ & X \ge 0, \end{aligned} \tag{2}$$

where $f_j(X)$ and $g_i(X)$ are real-valued functions and $c_i \in \mathbb{R}$, $\forall j = 1, 2, ..., k$ and $\forall i = 1, 2, ..., m$.

Definition 3.1.4 (Cristofari et al., 2024). Let $\mathbb{S}$ be the set of all feasible solutions of problem (2), and let $X, X^* \in \mathbb{S}$. Then, X^* is said to be a Pareto optimal solution (POS) for problem (2) if and only if there does not exist $X \in \mathbb{S}$ such that $f_j(X^*) \le f_j(X)$, $\forall j = 1, 2, \ldots, k_1$, and $\exists j \in \{1, 2, \ldots, k_1\}$ such that $f_j(X^*) < f_j(X)$.

Theorem 3.1.1 (Xu & Cai, 2012; Xu & Yager, 2006).
Let $\tilde{I_j} = (\mu_{\tilde{I_j}}, \nu_{\tilde{I_j}}), j = 1, 2, ..., k$ be a collection of intuitionistic fuzzy values, and let $w_1, w_2, ..., w_k$ be the corresponding weights, where $w_j \in (0, 1)$ and $\sum_{j=1}^{k} w_j = 1$. Then, the aggregated value using the intuitionistic fuzzy weighted geometric (IFWG) operator is given by

$$\text{IFWG}_w(\tilde{I_1}, \tilde{I_2}, ..., \tilde{I_k}) = \left(\prod_{j=1}^{k} \mu_{\tilde{I_j}}^{w_j}, 1 - \prod_{j=1}^{k} (1 - \nu_{\tilde{I_j}})^{w_j} \right) \tag{3}$$

3.2 The Solution Method

Most existing studies consider the optimistic variant of the problem (Kis et al., 2021), paying little attention to alternative perspectives in the solution process. However, this consideration has its own limitations in addressing practical problems. Due to the inconsistent nature of human judgment, DMs may deviate from their initial standpoint after evaluating the obtained solution against the intended goals. Consequently, existing methods overlook this important aspect of DM judgment and insights. Therefore, identifying the influence of optimistic, pessimistic, and mixed perspectives is highly valuable for obtaining consistent and robust solutions to MOO problems (Chen et al., 2023; Mahajan & Gupta, 2021b). The main difference among these three perspectives arises from the choice of violation and tolerance values used to determine the non-membership degrees, while the membership function remains identical across all cases.

In an IFE, parameter values, aspiration levels, coefficients of the objectives and constraints, as well as the equalities and inequalities in the model, are flexible. Consequently, the maximum and minimum allowable values of the constraints and objectives can vary.

To incorporate flexibility in the constraints, a violation parameter ℓ_i is associated with the ith constraint, $i = 1, 2, \ldots, m_1$, as specified by the DMs. For constraints of the form $\le$, the upper bound c_i is relaxed to $c_i + \lambda(\ell_i)$ in the solution procedure (Tsegaye et al., 2021).

The membership and non-membership functions for each objective are described based on the difference between the maximum U_j and minimum L_j achievable goals, which are identified using a table of extreme solutions. Then, the tolerance variables δ_j and ζ_j are obtained by using

$$\delta_j = \lambda(U_j - L_j) \text{ and } \zeta_j = \lambda(U_j - (L_j - \delta_j)) = \delta_j(1 + \lambda) \text{ where}$$
$$L_j = \min\{f_j(X)\}, U_j = \max\{f_j(X)\}, \text{and } \lambda \in (0, 1), j = 1, 2, ..., k.$$

The membership and non-membership functions are generally characterized by nonlinear behavior due to instantaneous variation at each solution point. Among nonlinear functions, the exponential function is preferable for the IFMOO problem because of its efficiency and flexibility in evaluating the marginal values of objectives and constraints (Ahmadini & Ahmad, 2021; Mahajan & Gupta, 2021a). Accordingly, exponential membership and non-membership functions are formulated to describe the optimistic, pessimistic, and mixed features and to obtain an efficient solution for the IFMOO problem.

3.2.1 The optimistic approach

In the optimistic approach, DM takes a liberal view of rejection (V. Singh et al., 2021). Therefore, the linear membership $(\mu_{U_j}(f_j(X)))$ and non-membership $(\nu_{U_j}(f_j(X)))$ functions for the jth objective function $f_j(X)$ of the maximization problem are respectively described as follows:

$$\mu_{U_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \le L_j \\ \frac{f_j(X) - L_j}{U_j - L_j}, & \text{if } L_j < f_j(X) < U_j \\ 1, & \text{if } f_j(X) \ge U_j, \end{cases} \tag{4}$$

and

$$\nu_{U_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \le L_j - \delta_j \\ \frac{U_j - f_j(X)}{U_j - (L_j - \delta_j)}, & \text{if } L_j - \delta_j < f_j(X) < U_j \\ 0, & \text{if } f_j(X) \ge U_j, \end{cases} \tag{5}$$

where δ_j is a tolerance value of the jth objective and defined as $\delta_j = \lambda(U_j - L_j)$ and $\lambda \in (0, 1)$, $\forall j = 1, 2, ..., k_1$. If $U_j = L_j$, then we define $\mu_{U_j}(f_j(X)) = 1$.

The respective exponential membership function $(\mu^E_{U_j}(f_j(X)))$ and non-membership function $(\nu^E_{U_j}(f_j(X)))$ for the jth objective respectively defined as

$$\mu^E_{U_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \le L_j \\ \frac{e^{-d_j((U_j - f_j(X))/(U_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}}, & \text{if } L_j < f_j(X) < U_j \\ 1, & \text{if } f_j(X) \ge U_j, \end{cases} \tag{6}$$

and

$$\nu^E_{U_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \le L_j - \delta_j \\ \frac{e^{-d_j((f_j(X) + \delta_j - L_j)/(U_j + \delta_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}}, & \text{if } L_j - \delta_j < f_j(X) < U_j \\ 0, & \text{if } f_j(X) \ge U_j. \end{cases} \tag{7}$$

where, d_j is the shape parameter.

The linear membership and non-membership functions for minimization objectives can be described as

$$\mu_{L_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \le L_j \\ \frac{U_j - f_j(X)}{U_j - L_j}, & \text{if } L_j < f_j(X) < U_j \\ 0, & \text{if } f_j(X) \ge U_j, \end{cases} \tag{8}$$

and

$$\nu_{L_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \le L_j \\ \frac{f_j(X) - L_j}{(U_j + \delta_j) - L_j}, & \text{if } L_j < f_j(X) < U_j + \delta_j \\ 1, & \text{if } f_j(X) \ge U_j + \delta_j, \end{cases} \tag{9}$$

where δ_j is a tolerance value of the jth objective and defined as $\delta_j = \lambda(U_j - L_j)$ and $\lambda \in (0, 1)$, $\forall j = k_1 + 1, k_1 + 2, ..., k$. If $U_j = L_j$, then we define $\mu_{L_j}(f_j(X)) = 1$.

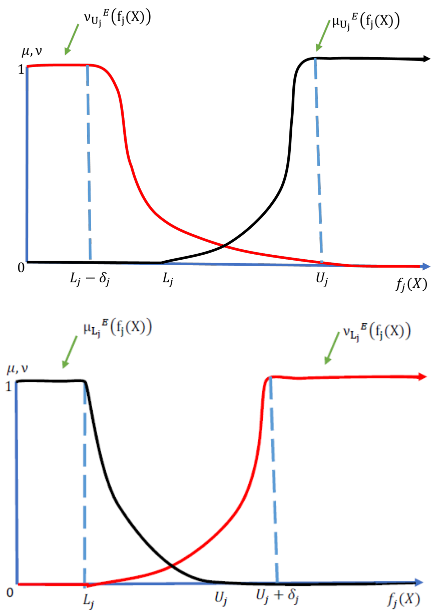
The corresponding exponential non-membership function $(\mu^E_{L_j}(f_j(X)))$ and non-membership function $(\nu^E_{L_j}(f_j(X)))$ for the jth objective respectively defined as

$$\mu^E_{L_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \le L_j \\ \frac{e^{-d_j((f_j(X) - L_j)/(U_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}}, & \text{if } L_j < f_j(X) < U_j \\ 1, & \text{if } f_j(X) \ge U_j, \end{cases} \tag{10}$$

and

$$\nu^E_{L_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \le L_j \\ \frac{e^{-d_j((U_j + \delta_j - f_j(X))/(U_j + \delta_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}}, & \text{if } L_j < f_j(X) < U_j + \delta_j \\ 0, & \text{if } f_j(X) \ge U_j + \delta_j. \end{cases} \tag{11}$$

Their general shape is shown in Figure 1.



(a) (b)

Figure 1: Exponential membership and non-membership functions for maximization (a) and minimization (b) objectives under the optimistic approach.

3.2.2 The pessimistic approach

In the pessimistic approach, the DM is presumably extra cautious about acceptance (V. Singh et al., 2021). The linear non-membership function $\nu_{U_j}(f_j(X))$ of the jth objective $f_j(X)$ under a pessimistic approach to the maximization problem is expressed as

$$\nu_{U_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j \\ \frac{(L_j+\delta_j)-f_j(X)}{(L_j+\delta_j)-L_j}, & \text{if } L_j < f_j(X) < L_j+\delta_j \\ 0, & \text{if } f_j(X) \geq L_j+\delta_j, \end{cases} \quad (12)$$

$\forall j = 1, 2, ..., k_1$.
The corresponding exponential non-membership function $\nu^E_{U_j}(f_j(X))$ is defined as

$$\nu^E_{U_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j \\ \frac{e^{-d_j((f_j(X)-L_j)/\delta_j)}-e^{-d_j}}{1-e^{-d_j}}, & \text{if } L_j < f_j(X) < L_j+\delta_j \\ 0, & \text{if } f_j(X) \geq L_j+\delta_j. \end{cases} \quad (13)$$

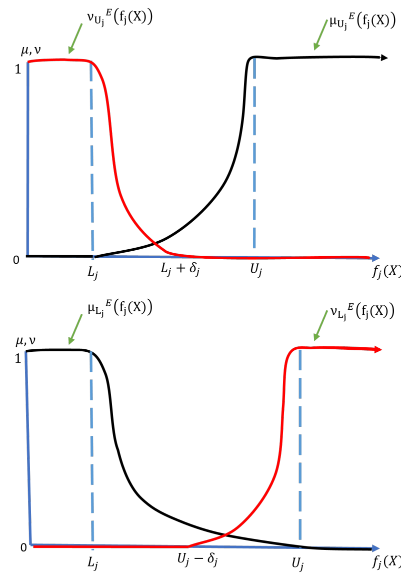
The linear non-membership function $\nu_{L_j}(f_j(X))$ of the jth objective $f_j(X)$ under a pessimistic approach to the minimization problem is expressed as

$$\nu_{L_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \leq U_j-\delta_j \\ \frac{f_j(X)-(U_j-\delta_j)}{U_j-(U_j-\delta_j)}, & \text{if } U_j-\delta_j < f_j(X) < U_j \\ 1, & \text{if } f_j(X) \geq U_j, \end{cases} \quad (14)$$

$\forall j = k_1+1, k_1+2, ..., k$.
The corresponding exponential non-membership function $\nu^E_{L_j}(f_j(X))$ is defined as

$$\nu^E_{L_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \leq U_j-\delta_j \\ \frac{e^{-d_j(U_j-f_j(X))/\delta_j)}-e^{-d_j}}{1-e^{-d_j}}, & \text{if } U_j-\delta_j < f_j(X) < U_j \\ 1, & \text{if } f_j(X) \geq U_j. \end{cases} \quad (15)$$

Their possible shape is shown in Figure 2.



(a) (b)

Figure 2: Exponential membership and non-membership functions for maximization (a) and minimization (b) objectives under a pessimistic approach.

3.2.2 The mixed approach

In a mixed approach, the DM is not flexible in rejecting and is not capable of extra acceptance (V. Singh & Yadav, 2018). The linear non-membership function $\nu_{U_j}(f_j(X)))$ of the jth objective function $f_j(X)$ to the maximization problem is defined as

$$\nu_{U_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j-\delta_j \\ \frac{(L_j+\zeta_j)-f_j(X)}{(L_j+\zeta_j)-(L_j-\delta_j)}, & \text{if } L_j-\delta_j < f_j(X) < L_j+\zeta_j \\ 0, & \text{if } f_j(X) \geq L_j+\zeta_j, \end{cases} \quad (16)$$

where δ_j and ζ_j are the tolerance variables of the jth objective and defined as $\delta_j = \lambda(U_j-L_j)$,
$\zeta_j = \delta_j(1+\lambda)$, $\lambda \in (0,1)$, $j = 1, 2, ..., k_1$.
The corresponding exponential non-membership function $\nu^E_{U_j}(f_j(X))$ is constructed as

$$\nu^E_{U_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j-\delta_j \\ \frac{e^{-d_j((f_j(X)-(L_j-\delta_j))/(\zeta_j+\delta_j))}-e^{-d_j}}{1-e^{-d_j}}, & \text{if } L_j-\delta_j < f_j(X) < L_j+\zeta_j \\ 0, & \text{if } f_j(X) \geq L_j+\zeta_j. \end{cases} \quad (17)$$

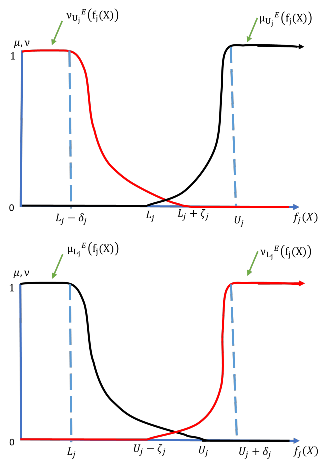
The linear non-membership function $\nu_{L_j}(f_j(X)))$ of the jth objective function $f_j(X)$ to the minimization problem is defined as

$$\nu_{L_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq U_j-\zeta_j \\ \frac{f_j(X)-(U_j-\zeta_j)}{(U_j+\delta_j)-(U_j-\zeta_j)}, & \text{if } U_j-\zeta_j < f_j(X) < U_j+\delta_j \\ 0, & \text{if } f_j(X) \geq U_j+\delta_j, \end{cases} \quad (18)$$

where δ_j and ζ_j are the tolerance variables of the jth objective and defined as $\delta_j = \lambda(U_j-L_j)$, $\zeta_j = \delta_j(1+\lambda)$, $\lambda \in (0,1)$, $j = k_1+1, k_1+2, ..., k$. The corresponding exponential non-membership function $\nu^E_{L_j}(f_j(X))$ is constructed as

$$\nu^E_{L_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \leq U_j-\zeta_j \\ \frac{e^{-d_j((U_j+\delta_j)-f_j(X))/(\zeta_j+\delta_j))}-e^{-d_j}}{1-e^{-d_j}}, & \text{if } U_j-\zeta_j < f_j(X) < U_j+\delta_j \\ 1, & \text{if } f_j(X) \geq U_j+\delta_j. \end{cases} \quad (19)$$

Their general shape is shown in Figure 3.



(a) (b)

Figure 3: Exponential membership and non-membership functions for maximization (a) and minimization (b) objectives under the mixed approach.

After converting an IFMOO problem into its equivalent crisp MOO problem using the accuracy function (S. K. Singh & Yadav, 2015a), the membership and non-membership functions are constructed based on the DMs' viewpoints. Accordingly, aggregation operators have been proposed (Mahajan & Gupta, 2021b; S. K. Singh & Yadav, 2015b; V. Singh & Yadav, 2018) to combine the membership and non-membership functions. However, such aggregation methods have certain limitations, as they fail to consider for both the satisfaction and dissatisfaction associated with all objectives. To address this limitation, the IFWG operator, previously applied to various multicriteria decision-making problems, is extended in this study to the MOO problem.

To formulate a single aggregation operator based on Theorem 3.1.1, a multiplicative combination is employed to integrate the independently defined degrees of acceptance and rejection. This approach helps to emphasize the interaction between the two degrees. Accordingly, the aggregation operator can be expressed as

$$\begin{aligned} Z(\mu,\nu) &= \left(\prod_{j=1}^{k}\mu_j^{w_j}\right)\times\left(1-(1-\prod_{j=1}^{k}(1-\nu_j)^{w_j})\right) \\ &= \left(\prod_{j=1}^{k}\mu_j^{w_j}\right)\times\left(\prod_{j=1}^{k}(1-\nu_j)^{w_j}\right) \end{aligned} \tag{20}$$

Therefore, the IFMOO problem (1) can be solved using an equivalent crisp model by employing the aggregation operator (20) as follows:

$$\begin{aligned} &\max \left(\prod_{j=1}^{k}\alpha_{o_j}^{w_j}\right)\times\left(\prod_{j=1}^{k}(1-\beta_{o_j})^{w_j}\right) \\ &\quad \text{subject to} \\ &\mu_{U_j}^{E}(f_j(X)) \geq \alpha_{o_j},\ j=1,2,\dots,k_1 \\ &\mu_{L_j}^{E}(f_j(X)) \geq \alpha_{o_j},\ j=k_1+1,k_1+2,\dots,k \\ &\nu_{U_j}^{E}(f_j(X)) \leq \beta_{o_j},\ j=1,2,\dots,k_1 \\ &\nu_{L_j}^{E}(f_j(X)) \leq \beta_{o_j},\ j=k_1+1,k_1+2,\dots,k \\ &0 \leq \alpha_{o_j}+\beta_{o_j} \leq 1,\ j=1,2,\dots,k_1,k_1+1,k_1+2,\dots,k \\ &0 \leq \beta_{o_j} \leq \alpha_{o_j},\ j=1,2,\dots,k_1,k_1+1,k_1+2,\dots,k \\ &g_i(X) \leq c_i+\lambda(\ell_i),\ i=1,2,\dots,m_1 \\ &X \geq 0, \end{aligned} \tag{21}$$

where w_j is the weight assigned to the jth objective such that $w_j \in (0,1)$ and $\sum_{j=1}^{k} w_j = 1$.

Problem (21) can be expressed based on the DM's perspectives, using the membership and non-membership functions constructed from Sections 3.2.1 to 3.2.3, as presented below.

From an optimistic viewpoint, problem (21) can be expressed as

$$\begin{aligned} &\max \left(\prod_{j=1}^{k}\alpha_{o_j}^{w_j}\right)\times\left(\prod_{j=1}^{k}(1-\beta_{o_j})^{w_j}\right) \\ &\quad \text{subject to} \\ &e^{d_j(1-((U_j-f_j(X))/(U_j-L_j)))}+\alpha_{o_j}(1-e^{d_j}) \geq 1,\ j=1,2,\dots,k_1 \\ &e^{d_j(1-((f_j(X)-L_j)/(U_j-L_j)))}+\alpha_{o_j}(1-e^{d_j}) \geq 1,\ j=k_1+1,k_1+2,\dots,k \\ &e^{d_j(1-((f_j(X)-(L_j-\delta_j))/(U_j-(L_j-\delta_j))))}+\beta_{o_j}(1-e^{d_j}) \leq 1,\ j=1,2,\dots,k_1 \\ &e^{d_j(1-(((U_j+\delta_j)-f_j(X))/((U_j+\delta_j)-L_j)))}+\beta_{o_j}(1-e^{d_j}) \leq 1,\ j=k_1+1,k_1+2,\dots,k \\ &0 \leq \alpha_{o_j}+\beta_{o_j} \leq 1,\ j=1,2,\dots,k_1,k_1+1,k_1+2,\dots,k \\ &0 \leq \beta_{o_j} \leq \alpha_{o_j},\ j=1,2,\dots,k_1,k_1+1,k_1+2,\dots,k \\ &g_i(X) \leq c_i+\lambda(\ell_i),\ i=1,2,\dots,m_1 \\ &X \geq 0. \end{aligned} \tag{22}$$

The pessimistic variant of problem (21) described as

$$\begin{aligned} &\max \left(\prod_{j=1}^{k}\alpha_{o_j}^{w_j}\right)\times\left(\prod_{j=1}^{k}(1-\beta_{o_j})^{w_j}\right) \\ &\quad \text{subject to} \\ &e^{d_j(1-((U_j-f_j(X))/(U_j-L_j)))}+\alpha_{o_j}(1-e^{d_j}) \geq 1,\ j=1,2,\dots,k_1 \\ &e^{d_j(1-((f_j(X)-L_j)/(U_j-L_j)))}+\alpha_{o_j}(1-e^{d_j}) \geq 1,\ j=k_1+1,k_1+2,\dots,k \\ &e^{d_j(1-((f_j(X)-L_j)/\delta_j))}+\beta_{o_j}(1-e^{d_j}) \leq 1,\ j=1,2,\dots,k_1 \\ &e^{d_j(1-((U_j-f_j(X))/\delta_j))}+\beta_{o_j}(1-e^{d_j}) \leq 1,\ j=k_1+1,k_1+2,\dots,k \\ &0 \leq \alpha_{o_j}+\beta_{o_j} \leq 1,\ j=1,2,\dots,k_1,k_1+1,k_1+2,\dots,k \\ &0 \leq \beta_{o_j} \leq \alpha_{o_j},\ j=1,2,\dots,k_1,k_1+1,k_1+2,\dots,k \\ &g_i(X) \leq c_i+\lambda(\ell_i),\ i=1,2,\dots,m_1 \\ &X \geq 0. \end{aligned} \tag{23}$$

For the mixed approach, problem (21) is expressed as

$$\begin{aligned} &\max \left(\prod_{j=1}^{k}\alpha_{o_j}^{w_j}\right)\times\left(\prod_{j=1}^{k}(1-\beta_{o_j})^{w_j}\right) \\ &\quad \text{subject to} \\ &e^{d_j(1-((U_j-f_j(X))/(U_j-L_j)))}+\alpha_{o_j}(1-e^{d_j}) \geq 1,\ j=1,2,\dots,k_1 \\ &e^{d_j(1-((f_j(X)-L_j)/(U_j-L_j)))}+\alpha_{o_j}(1-e^{d_j}) \geq 1,\ j=k_1+1,k_1+2,\dots,k \\ &e^{d_j(1-((f_j(X)-(L_j-\delta_j))/(\delta_j+\zeta_j)))}+\beta_{o_j}(1-e^{d_j}) \leq 1,\ j=1,2,\dots,k_1 \\ &e^{d_j(1-(((U_j+\delta_j)-f_j(X))/(\delta_j+\zeta_j)))}+\beta_{o_j}(1-e^{d_j}) \leq 1,\ j=k_1+1,k_1+2,\dots,k \\ &0 \leq \alpha_{o_j}+\beta_{o_j} \leq 1,\ j=1,2,\dots,k_1,k_1+1,k_1+2,\dots,k \\ &0 \leq \beta_{o_j} \leq \alpha_{o_j},\ j=1,2,\dots,k_1,k_1+1,k_1+2,\dots,k \\ &g_i(X) \leq c_i+\lambda(\ell_i),\ i=1,2,\dots,m_1 \\ &X \geq 0. \end{aligned} \tag{24}$$

Theorem 3.2.1. A unique optimal solution (X^*,α^*,β^*) of problem (22) corresponds to a POS X^* of problem (2), where $\alpha^* = (\alpha^*_{o_1},\alpha^*_{o_2},\dots,\alpha^*_{o_{k_1}})$

and $\beta^* = (\beta^*_{o_1}, \beta^*_{o_2}, ..., \beta^*_{o_{k_1}})$.

Proof. Since

$$Z(\alpha,\beta) = \left(\prod_{j=1}^{k_1} \alpha_{o_j}^{w_j}\right) \times \left(\prod_{j=1}^{k_1} (1-\beta_{o_j})^{w_j}\right)$$

Let (X^*, α^*, β^*) be a unique solution of problem (22). Then

$$Z(\alpha^*,\beta^*) > Z(\alpha,\beta), \quad \forall (X,\alpha,\beta) \text{ in the feasible space of (22).}$$

Suppose X^* is not a POS of problem (2). Then, by Definition 3.1.4, $\exists \hat{X}$ in the feasible space of (2) such that

$$f_j(\hat{X}) \geq f_j(X^*), \forall j = 1,2,\ldots,k_1 \text{ and } f_j(\hat{X}) > f_j(X^*), \text{for at least one} j \in \{1,2,...,k_1\} \ (i)$$

Thus,

$$\frac{f_j(\hat{X}) - L_j}{U_j - L_j} \geq \frac{f_j(X^*) - L_j}{U_j - L_j}, \quad \forall j = 1,2,...,k_1,$$

and

$$\frac{f_j(\hat{X}) - L_j}{U_j - L_j} > \frac{f_j(X^*) - L_j}{U_j - L_j} \quad \text{for at least one } j \in \{1,2,...,k_1\}.$$

Since $d_j > 0$,

$$\Longrightarrow \frac{e^{-d_j((U_j - f_j(\hat{X}))/(U_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}} \geq \frac{e^{-d_j((U_j - f_j(X^*))/(U_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}}, \quad \forall j \in \{1,2,...,k_1\},$$

and

$$\frac{e^{-d_j((U_j - f_j(\hat{X}))/(U_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}} > \frac{e^{-d_j((U_j - f_j(X^*))/(U_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}},$$

for at least one $j \in \{1,2,...,k_1\}$. (ii)

Similarly, for non-membership functions:

$$\frac{U_j - f_j(\hat{X})}{U_j - (L_j - \delta_j)} \leq \frac{U_j - f_j(X^*)}{U_j - (L_j - \delta_j)}, \forall j \in \{1,2,...,k_1\},$$

and

$$\frac{U_j - f_j(\hat{X})}{U_j - (L_j - \delta_j)} < \frac{U_j - f_j(X^*)}{U_j - (L_j - \delta_j)}, \quad \text{for at least one } j \in \{1,2,...,k_1\},$$

$$\Longrightarrow \frac{e^{-d_j((f_j(\hat{X}) + \delta_j - L_j)/(U_j + \delta_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}} \leq \frac{e^{-d_j((f_j(X^*) + \delta_j - L_j)/(U_j + \delta_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}},$$

$\forall j \in \{1,2,...,k_1\}$, and

$$\frac{e^{-d_j((f_j(\hat{X}) + \delta_j - L_j)/(U_j + \delta_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}} < \frac{e^{-d_j((f_j(X^*) + \delta_j - L_j)/(U_j + \delta_j - L_j))} - e^{-d_j}}{1 - e^{-d_j}},$$

for at least one $j \in \{1,2,...,k_1\}$. (iii)

Define

$$\hat{\alpha}_{o_j} = \frac{e^{-d_j\left(\frac{U_j - f_j(\hat{X})}{U_j - L_j}\right)} - e^{-d_j}}{1 - e^{-d_j}},$$

$$\alpha^*_{o_j} = \frac{e^{-d_j\left(\frac{U_j - f_j(X^*)}{U_j - L_j}\right)} - e^{-d_j}}{1 - e^{-d_j}},$$

$$\hat{\beta}_{o_j} = \frac{e^{-d_j\left(\frac{f_j(\hat{X}) + \delta_j - L_j}{U_j + \delta_j - L_j}\right)} - e^{-d_j}}{1 - e^{-d_j}},$$

$$\beta^*_{o_j} = \frac{e^{-d_j\left(\frac{f_j(X^*) + \delta_j - L_j}{U_j + \delta_j - L_j}\right)} - e^{-d_j}}{1 - e^{-d_j}}.$$

Then, using (ii) and (iii), we get

$$\hat{\alpha}_{o_j} \geq \alpha^*_{o_j}, \forall j \in \{1,2,...,k_1\} \quad \text{and} \quad \hat{\alpha}_{o_j} > \alpha^*_{o_j} \text{ for at least one } j \in \{1,2,...,k_1\}. \quad \text{(iv)}$$

$$\hat{\beta}_{o_j} \leq \beta^*_{o_j} \Rightarrow 1 - \hat{\beta}_{o_j} \geq 1 - \beta^*_{o_j}, \ \forall j \in \{1,2,...,k_1\},$$
$$\hat{\beta}_{o_j} < \beta^*_{o_j} \Rightarrow 1 - \hat{\beta}_{o_j} > 1 - \beta^*_{o_j}, \text{ for at least one } j \in \{1,2,...,k_1\}. \quad \text{(v)}$$

Now, using (iv), we get

$$\prod_{j=1}^{k_1} \hat{\alpha}_{o_j}^{w_j} > \prod_{j=1}^{k_1} (\alpha^*_{o_j})^{w_j}. \quad \text{(vi)}$$

Using (v), we obtain

$$\prod_{j=1}^{k_1} (1 - \hat{\beta}_{o_j})^{w_j} > \prod_{j=1}^{k_1} (1 - \beta^*_{o_j})^{w_j}. \quad \text{(vii)}$$

Finally, from (vi) and (vii), we conclude:

$$\prod_{j=1}^{k_1} \hat{\alpha}_{o_j}^{w_j} \prod_{j=1}^{k_1} (1 - \hat{\beta}_{o_j})^{w_j} > \prod_{j=1}^{k_1} (\alpha^*_{o_j})^{w_j} \prod_{j=1}^{k_1} (1 - \beta^*_{o_j})^{w_j}$$

$$\Rightarrow Z(\hat{\alpha}, \hat{\beta}) > Z(\alpha^*, \beta^*),$$

which contradicts the optimality of (X^*, α^*, β^*) for problem (22). Hence, no such $\hat{X}$ exists, and therefore X^* is a POS of problem (2). □

The theorem can be proved similarly for the remaining two perspectives of the DMs and for minimization objectives.

The overall solution procedure of the proposed method for solving an IFMOO problem can be summarized as follows:

Step 1. Formulate the IFMOO problem (1).

Step 2. Transform the IFMOO problem into the equivalent crisp MOOP (2) by employing the accuracy function.

Step 3. Solve each objective function independently and construct the pay-off table to determine the lower and upper bounds, denoted by L_j and U_j, respectively, for each objective function $f_j(X)$, where $j = 1,2,...,k$.

Step 4. Construct the exponential membership and non-membership functions corresponding to each objective function under the optimistic, pessimistic, and mixed approaches using the tolerance parameters δ_j and ζ_j, where $j = 1,2,...,k$.

Step 5. Relax or reduce the ≤ type and ≥ type constraints by employing the assigned violation parameter l_i in the forms $c_i + \lambda(l_i)$ and $c_i - \lambda(l_i)$, respectively, where $\lambda \in (0,1)$ and $i = 1,2,...,m$.

Step 6. Develop and solve the problem according to the DM's preference under the optimistic (22) or pessimistic (23) or mixed (24) model.

Step 7. If the obtained solution satisfies the DM, then terminate the solution procedure. Otherwise, reformulate the problem and repeat the process until a satisfactory solution is obtained.

4 Comparative Analysis

To verify the effectiveness of the proposed method, a comparative analysis was conducted using the approach proposed by V. Singh and Yadav (2018). The manufacturing problem presented in their study was employed for this purpose. Accordingly, the transformed equivalent crisp problem is

$$
\begin{aligned}
&\max\ f_1(X) = 7.5x_1 + 10.125x_2 + 8x_3,\\
&\min\ f_2(X) = 2.9375x_1 + 3.8750x_2 + 5.1250x_3\\
&\qquad\qquad \text{subject to}\\
&2.9375x_1 + 2.0625x_2 + 2.9375x_3 \leq 328.125,\\
&3.875x_1 + 2.9375x_2 + 2.0625x_3 \leq 355.625,\\
&2.0625x_1 + 2.9375x_2 + 2.9375x_3 \geq 355,\\
&X = (x_1, x_2, x_3) \geq 0.
\end{aligned}
\tag{25}
$$

By solving each objective independently, subject to the constraints, the ideal solutions and the extreme values of the objectives are identified. Accordingly, $X_1 = (0, 84.09, 52.66)$ and $X_2 = (0, 120.85, 0)$ which give $L_1 = 1223.62, U_1 = 1272.69$, and $L_2 = 468.30, U_2 = 595.73$.

For an optimistic DM, problem (25) is transformed into a single-objective problem of the form (26) using model (22), by assigning equal weights to the objectives and considering violation parameters of 25 and 20 units for the first and second constraints, respectively.

$$
\begin{aligned}
&\max Z = \alpha_1^{0.5}\alpha_2^{0.5}(1-\beta_1)^{0.5}(1-\beta_2)^{0.5}\\
&\qquad\qquad \text{subject to}\\
&e^{0.2(1-((1272.69-(7.5x_1+10.125x_2+8x_3))/(1272.69-1223.62)))} + \alpha_1(1-e^{0.2}) \geq 1,\\
&e^{0.2(1-((2.9375x_1+3.8750x_2+5.1250x_3-468.30)/(595.73-468.30)))} + \alpha_2(1-e^{0.2}) \geq 1,\\
&e^{0.2(1-((7.5x_1+10.125x_2+8x_3-(1223.62-(\lambda(1272.69-1223.62))))/(1272.69-(1223.62-(\lambda(1272.69-1223.62))))))} +\\
&\beta_1(1-e^{0.2}) \leq 1,\\
&e^{0.2(1-(((595.73+(\lambda(595.73-468.30)))-(2.9375x_1+3.8750x_2+5.1250x_3))/((595.73+(\lambda(595.73-468.30)))-468.30)))} +\\
&\beta_2(1-e^{0.2}) \leq 1,\\
&2.0625x_1 + 3.8750x_2 + 2.9375x_3 \leq 333.125 + \lambda(25),\\
&3.8750x_1 + 2.0625x_2 + 2.0625x_3 \leq 365.625 + \lambda(20),\\
&2.9375x_1 + 2.0625x_2 + 2.9375x_3 \geq 360,\\
&0 \leq \alpha_j + \beta_j \leq 1, \quad j = 1, 2\\
&0 \leq \alpha_j \leq 1, \quad j = 1, 2\\
&0 \leq \beta_j \leq 1, \quad j = 1, 2\\
&x_1, x_2, x_3 \geq 0.
\end{aligned}
\tag{26}
$$

Solving this problem, using LINGO (LINDO Systems Inc., 2017) version 21, the solution is presented in Table 1.

Table 1: Solutions of problem (26) under different λ values.

λ	X	$f_1(X)$	$f_2(X)$	$g_1(X)$	$g_2(X)$	$g_3(X)$
0.3	$(0.00, 121.49, 3.30)$	1248.50	482.56	257.33	361.63	363.63
0.4	$(0.00, 123.79, 0.00)$	1253.35	479.68	255.31	363.62	363.62
0.5	$(0.00, 124.46, 0.00)$	1260.24	482.31	256.72	365.62	365.62
0.6	$(0.00, 125.14, 0.00)$	1267.13	484.95	258.12	367.62	367.62
0.7	$(0.00, 125.70, 0.00)$	1272.69	487.08	259.25	369.24	369.24
0.8	$(0.00, 125.70, 0.00)$	1272.69	487.08	259.25	369.24	369.24

As λ increases, $f_1(X)$ improves while $f_2(X)$ reaches its lowest value around $\lambda = 0.4$ and then slightly worsens for larger values of λ. This shows a trade-off, where increasing emphasis shifts from minimizing $f_2(X)$ toward maximizing $f_1(X)$. The solution becomes stable for larger λ.

Similarly, by reformulating problem (25) for pessimistic and mixed DM perspectives using models (23) and (24), respectively, we obtain the following solutions presented in Tables 2 and 3, respectively.

Table 2: Solutions of Problem (25) under the pessimistic perspective.

λ	X	$f_1(X)$	$f_2(X)$	$g_1(X)$	$g_2(X)$	$g_3(X)$
0.3	$(0.00, 114.81, 11.82)$	1256.98	505.45	271.51	361.62	371.97
0.4	$(0.00, 118.60, 7.39)$	1259.93	497.44	266.32	363.62	370.09
0.5	$(0.00, 122.39, 2.96)$	1262.88	489.44	261.13	365.63	368.22
0.6	$(2.02, 118.03, 6.35)$	1260.97	495.83	268.01	367.62	369.52
0.7	$(2.91, 118.91, 4.40)$	1260.91	491.84	266.70	369.62	409.67
0.8	$(3.47, 118.96, 4.23)$	1264.38	492.86	267.98	371.62	369.04

For the pessimistic variants of the problem, as λ increases, $f_1(X)$ improves while $f_2(X)$ remains relatively stable in the mid-range of λ, with only minor fluctuations in both objectives.

Table 3: Solutions of Problem (25) under the mixed perspective.

λ	X	$f_1(X)$	$f_2(X)$	$g_1(X)$	$g_2(X)$	$g_3(X)$
0.3	$(0.00, 114.81, 11.82)$	1256.98	505.45	271.51	361.62	371.97
0.4	$(0.00, 118.60, 7.39)$	1259.93	497.44	266.32	363.62	370.09
0.5	$(0.00, 119.26, 7.42)$	1262.88	500.15	267.76	365.62	372.12
0.6	$(0.00, 125.15, 0.00)$	1267.13	484.95	258.12	367.62	367.62
0.7	$(0.00, 125.49, 0.00)$	1270.58	486.27	258.82	368.62	368.62
0.8	$(0.00, 125.56, 0.00)$	1271.13	486.54	258.96	368.82	368.82

For the mixed variants of the problem, as λ increases, the solution steadily improves $f_1(X)$, while $f_2(X)$ stabilizes after an initial fluctuation. Around $\lambda = 0.6$, the solution minimizes $f_2(X)$.

Table 4: Comparison of the proposed method with existing approach.

Variant	Proposed method ($\lambda = 0.3$ to 0.8)	Existing method (from $t = 1$ to 5, under two reference conditions)
Optimistic	$f_1(X)$ maximized within the range 1248.50 to 1272.69, $f_2(X)$ minimized within the range 487.08 to 479.68	$f_1(X)$ maximized within the range 1244.91 to 1248.62, $f_2(X)$ minimized within the range 530.80 to 520.33
Pessimistic	$f_1(X)$ maximized within the range 1256.98 to 1264.38, $f_2(X)$ minimized within the range 505.45 to 489.44	$f_1(X)$ maximized within the range 1248.63 to 1251.92, $f_2(X)$ minimized within the range 539.37 to 530.7
Mixed	$f_1(X)$ maximized within the range 1256.98 to 1271.13, $f_2(X)$ minimized within the range 505.45 to 484.95	$f_1(X) = 1248.77$ (maximized), $f_2(X) = 530.66$ (minimized)

The proposed method dominates the existing approaches across all decision-making variants by achieving higher values of $f_1(X)$ and lower values of $f_2(X)$ under different values of $\lambda \in (0, 1)$. It shows a significant reduction in $f_2(X)$, indicating improved minimization performance. Generally, the results demonstrate that the proposed model provides more efficient and balanced trade-off between the two conflicting objectives.

5 Model Application

5.1 Description of the Study Area

The farming site is in Abeshge district, Gurage Zone, central Ethiopia, between 8°19′–8°45′ N latitude and 37°45′–38°7′ E longitude (Nasir & Hundie, 2014). Mean annual temperatures range from 18°C to 28.3°C, with rainfall of 801–1400 mm, mostly during the Kiremt season (Dessie et al., 2017). The soil is sandy loam, with a pH of 6.40–6.92.

Farming is mainly rainfed due to limited irrigation, though a few seasonal rivers support perennial crops like mangoes and bananas along their banks. *Teff* (Eragrostis tef), maize (Zea mays), pepper (Capsicum annuum), chickpea (Cicer arietinum), bean (Phaseolus vulgaris), and sorghum (Sorghum bicolor) are the most widely cultivated crops in the area. These crops dominate the farming pattern, accounting for about 85% of the cropped area in the district.

The study area was chosen due to its high crop production potential and the availability of accessible agricultural data. The considered LSF has detailed information on existing cropping patterns, which makes the farming site suitable for empirical analysis. Furthermore, the study area represents the dominant farming system of the area, allowing the findings to be relevant to smallholder farmers in the area. This LSF practices rainfed farming on 1,033ha of land in Gefersa kebele. The location map of the study area is shown in Figure 4.

The existing farming pattern mainly targeted on the achievement of maximum production. The required input allocation to the crops are mainly determined by experience, even though they rarely apply the advice and paradigm of the developmental agents and extension workers in the district.

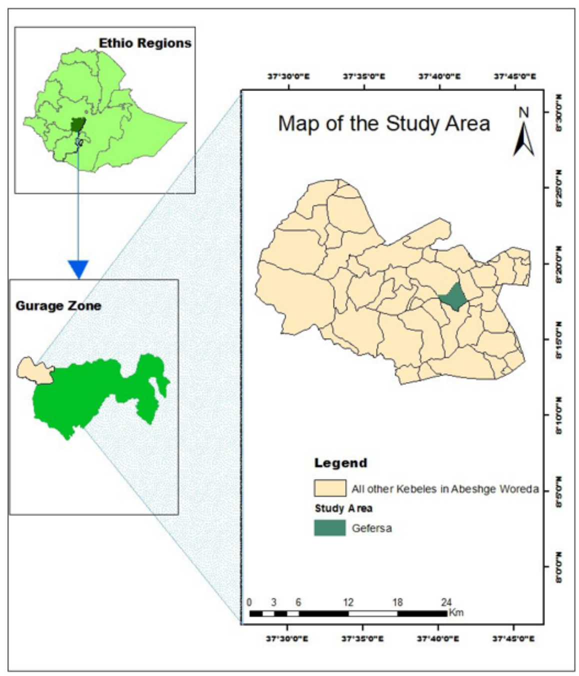


Figure 4: Map of the study area.

5.2 Methods of Data Collection and Analysis

A stratified purposive sampling method was employed to intentionally select the large-scale farm (LSF) site based on its suitability for the study. The selection criteria included the availability of reliable and accessible data, representative farming practices, adequate capital investment, and sufficient manpower. These characteristics enabled the farm to effectively represent the target study population and provide meaningful insights into the research objectives.

The data were collected from farmers, managers, and workers of the LSF, extension experts, development agents, the zonal rural development office, meteorological stations, and marketing agencies through surveys, questionnaires, and key informant interviews.

The dataset incorporates environmental, climatic, hydrological, agricultural, and socio-economic factors. Hydrological data cover effective rainfall and crop water demand. Effective rainfall was calculated using the dependable rain (FAO/AGLW formula) method Allen et al., 1998, while crop water demand was estimated with the Penman-Monteith equation (Allen et al., 1998; Smith, 1992) using crop evapotranspiration.

Crop data such as rooting depth, crop coefficient, critical depletion, yield response, crop height, and crop calendar were obtained from FAO Manual 56 (Allen et al., 1998) and related literature.
Socio-economic data include crop market prices, labor and machinery costs, fertilizer, herbicide, and pesticide dosages and prices, and land resource information.

The gathered data from different sources for a specific parameter are arranged into five groups, viz., the extreme minimum, minimum, median, maximum, and extreme maximum values, based on the level of their deviation from the median value. The values less than the median value are arranged into the extreme minimum and minimum values. Specifically, the average of the highly deviated values from the median value is taken as the extreme minimum value, while the average of the relatively less deviated values from the median is considered as the minimum value. In the same manner, the values greater than the median value are arranged into the maximum and extreme maximum values in the agricultural production problem. The average of the median values of a parameter is taken as the mean value.
Using this principle, the aggregated values are used to construct triangular IFNs to represent the APP problem more realistically.

In this study, the last twelve years climatic data from 2013 to 2024 were gathered from Emdbir meteorological station, with an altitude of $2082m$, latitude 8.13°N and longitude 37.93°E.
In order to handle the current extreme variation in the cost of input resources for crop cultivation and crop prices in the country, we have employed closely related data for the proposed study. Accordingly, the average data from 2022 to 2024 of the existing cropping schemes, crop yield (FAO, 2022, 2023), labor force, crop prices, cost of seeds, and other related expenditures for cultivation and agricultural input resources are presented in Tables 5 to 9.

Crop data are presented in Supplementary Materials S.7 and S.8. These, along with the soil data in Supplementary Material S.9, are used to calculate crop water requirements.
The considered crops are denoted as c_1 for maize, c_2 for *teff*, c_3 for sorghum, c_4 for pepper, c_5 for chickpea, c_6 for bean.

In Table 5, F_{1c_n} and F_{2c_n} represent the fertilizers required for crop c_n, where $n = 1, 2, 3, 4, 5, 6$, in the first and second rounds, respectively. Accordingly, F_{1c_n} denotes NPS, while F_{2c_n} represents Urea. Similarly, H_{1c_n} and H_{2c_n} represent the required herbicides for crop c_n in the first and second rounds, respectively. The pesticide required for crop c_n is denoted by P_{c_n}. Thus, the following symbols are used to denote different herbicides and pesticides: H_{2c_1} and H_{2c_3} for Atrazine, H_{1c_2} for 2,4-D, H_{2c_2} for Pallas 45 OD, H_{1c_4} for Glyphosate, H_{1c_5} and H_{2c_6} for Pendimethalin, H_{c_6} for S-metolachlor, P_{c_1} for Diazinon 60 EC, P_{c_2} for Dimethoate, P_{c_3} for Karate 5 EC, P_{c_4} for Ethiozinon 60 EC, P_{c_5} for Highway 50 EC, and P_{c_6} for Profit.

Table 5: Usage of fertilizer, herbicide and pesticide.

Crop	F_{1c_n} (kg/ha)	F_{2c_n} (kg/ha)	H_{1c_n} (L/ha)	H_{2c_n} (L/ha)	P_{c_n} (L/ha)
maize (c_1)	$\widetilde{100}$	$\widetilde{125}$	-	$\widetilde{1}$	$\widetilde{1}$
teff (c_2)	$\widetilde{100}$	$\widetilde{100}$	$\widetilde{1}$	$\widetilde{0.5}$	$\widetilde{1}$
sorghum (c_3)	$\widetilde{100}$	$\widetilde{100}$	-	$\widetilde{1}$	$\widetilde{1}$
pepper (c_4)	$\widetilde{100}$	$\widetilde{200}$	$\widetilde{1.5}$	-	$\widetilde{1.5}$
chickpea (c_5)	$\widetilde{100}$	-	$\widetilde{1}$	-	$\widetilde{1}$
bean (c_6)	$\widetilde{100}$	-	$\widetilde{1}$	$\widetilde{0.75}$	$\widetilde{1}$

The IFN in Table 5 describe the following numbers.

$\widetilde{200} = \langle 196, 198, 208;\ 194, 198, 210 \rangle$,
$\widetilde{125} = \langle 123, 125, 127;\ 122, 125, 128 \rangle$,
$\widetilde{100} = \langle 97, 99, 105;\ 95.5, 99, 106.5 \rangle$,
$\widetilde{2.5} = \langle 2, 2.6, 2.8;\ 1.9, 2.8, 2.9 \rangle$,
$\widetilde{0.5} = \langle 0.2, 0.5, 0.7;\ 0.1, 0.5, 1 \rangle$.
$\widetilde{1.5} = \langle 1.1, 1.6, 1.7;\ 1, 1.6, 1.8 \rangle$,
$\widetilde{1} = \langle 0.75, 1, 1.25;\ 0.5, 1, 1.5 \rangle$,
$\widetilde{0.75} = \langle 0.6, 0.7, 1;\ 0.3, 0.7, 1.3 \rangle$,
$\widetilde{2} = \langle 1.9, 2, 2.1;\ 1.5, 2, 2.5 \rangle$

The costs of cultivation, including plowing, threshing, land rent, seed, labor, herbicides and pesticides, and fertilizers, are provided in Supplementary Materials S.2, S.3, S.4, S.11, and S.12, respectively.
The overall cost of the cultivation of each crop is presented in Table 6.

Table 6: Cost of cultivation (I_{c_n}) and crop yield (Y_{c_n}).

Crop	I_{c_n} (ETB/ha)	Y_{c_n} (qtl/ha)
maize (c_1)	$\langle 35623, 35627, 35628; 35620, 35627, 35629 \rangle$	$\langle 50, 54, 62; 47, 54, 65 \rangle$
teff (c_2)	$\langle 35698, 35705, 35714; 35689, 35705, 35719 \rangle$	$\langle 12.5, 13.5, 16.5; 12, 13.5, 17 \rangle$
sorghum (c_3)	$\langle 26838, 26845, 26854; 26829, 26845, 26859 \rangle$	$\langle 23, 24.5, 27.5; 22.5, 24.5, 29 \rangle$
pepper (c_4)	$\langle 41113, 41120, 41129; 41104, 41120, 41134 \rangle$	$\langle 11, 13.5, 14; 10, 13.5, 16 \rangle$
chickpea (c_5)	$\langle 20655, 20659, 20666; 20649, 20659, 20674 \rangle$	$\langle 16.5, 17.5, 20; 15.5, 17.5, 22 \rangle$
bean (c_6)	$\langle 25124, 25131, 25140; 25115, 25131, 25145 \rangle$	$\langle 21, 23.5, 24; 20, 23.5, 25 \rangle$

The climatic data are used to calculate the crop water requirements, which are presented in Supplementary Material S.10.

Table 7: Seed (E_{c_n}), labor (B_{c_n}) and water requirement (W_{c_n}).

Crop	E_{c_n} (kg/ha)	B_{c_n} (md/ha)	W_{c_n} (m^3/ha)
maize (c_1)	$\langle 22, 26, 25; 17, 26, 27 \rangle$	$\langle 80, 86, 89; 75, 86, 92 \rangle$	$\langle 3570, 3610, 3625; 3565, 3610, 3648 \rangle$
teff (c_2)	$\langle 29, 30, 30.5; 28.5, 30, 32 \rangle$	$\langle 70, 76, 80; 64, 76, 82 \rangle$	$\langle 2580, 2625, 2640; 2564, 2625, 2660 \rangle$
sorghum (c_3)	$\langle 14, 14.5, 16.5; 13.5, 14.5, 18 \rangle$	$\langle 35, 41, 45; 30, 41, 46 \rangle$	$\langle 3240, 3249, 3252; 3225, 3249, 3255 \rangle$
pepper (c_4)	$\langle 15, 15.5, 17.5; 14.5, 15.5, 19 \rangle$	$\langle 97, 99, 105; 94, 99, 108 \rangle$	$\langle 4750, 4766, 4770; 4705, 4766, 4783 \rangle$
chickpea (c_5)	$\langle 26, 28, 30; 25, 28, 31 \rangle$	$\langle 25, 31, 35; 20, 31, 36 \rangle$	$\langle 2515, 2540, 2560; 2505, 2540, 2572 \rangle$
bean (c_6)	$\langle 19, 21, 23; 18, 21, 24 \rangle$	$\langle 47, 51, 52; 45, 51, 55 \rangle$	$\langle 2355, 2360, 2386; 2347, 2360, 2400 \rangle$

The farming machines used in the area are a tractor and a combine harvester. Machine hours for plowing are considered until the land is ready for sowing; details are in Supplementary Material S.1. The required total machine hours for each crop are shown in Table 8.

Table 8: Machine hours requirement (M_{cn}) and profit (N_{cn}).

Crop	M_{cn} (hr/ha)	N_{cn} (ETB/ha)
maize (c_1)	$\langle 1.83, 2.167, 2.33; 1.42, 2.167, 2.42 \rangle$	$\langle 33117, 33123, 33136; 33109, 33123, 33138 \rangle$
teff (c_2)	$\langle 1.83, 2.167, 2.33; 1.42, 2.167, 2.42 \rangle$	$\langle 13293, 13295, 13297; 13290, 13295, 13300 \rangle$
sorghum (c_3)	$\langle 1.75, 2, 2.42; 1.5, 2, 2.5 \rangle$	$\langle 18148, 18154, 18165; 18141, 18154, 18170 \rangle$
pepper (c_4)	$\langle 1.75, 2, 2.42; 1.5, 2, 2.5 \rangle$	$\langle 192873, 192881, 192888; 192862, 192881, 192893 \rangle$
chickpea (c_5)	$\langle 1.83, 2.167, 2.33; 1.42, 2.167, 2.42 \rangle$	$\langle 25830, 25842, 25849; 25820, 25842, 25853 \rangle$
bean (c_6)	$\langle 1.75, 2, 2.42; 1.5, 2, 2.5 \rangle$	$\langle 16262, 16269, 16278; 16253, 16269, 16283 \rangle$

The cropland allocation in the existing situation during 2020 - 2024 is given in the Supplementary Material S.5. Accordingly, the average land allocation of the existing system and the attainability and non-attainability degrees of the intended objectives are depicted in Table 9. Where x_1, x_2, x_3, x_4, x_5, and x_6 denote the land area allocated to maize, *teff*, sorghum, pepper, chickpea, and beans, respectively. The functions $f_1(X)$, $f_2(X)$, $f_3(X)$, and $f_4(X)$, respectively, represent the yield, profit, cost, and labor objectives.

Table 9: Existing farm pattern.

$X = (x_1, x_2, x_3, x_4, x_5, x_6)$		$f_1(X)$	$f_2(X)$	$f_3(X)$	$f_4(X)$
(221.02, 141.41, 401.32, 106.42, 62.29, 121.57)		29470.15	41510180	31413470	64160
	μ	0.81	0.20	0.17	0.20
	ν	0.19	0.28	0.30	0.28

Some mathematical software is employed to solve the considered APP problem and assist in analyzing the results of the study. CROPWAT 8.0 software (Smith, 1992) is used to generate and analyze the water requirement of crops, and LINGO (LINDO Systems Inc., 2017) is used to solve complex mathematical problems.

5.3 Problem Formulation

As a result of the inconsistent nature and imprecision of the pertained agricultural data, crop planning rests under the influence of risk and uncertainty (Luo et al., 2023). So the parameters in the APP problem are described as IFN. Thus, the objectives and constraints of the intended problem are expressed as intuitionistic fuzzy functions. Accordingly, IFMOO model is employed for comprehensive and efficacious investigation.

The *Kiremt* season is the widely practiced cropping season in the study area. Crops such as maize, pepper, and sorghum are sown during the *Belg* season and harvested by the end of *Kiremt*. *Teff* and beans are sown in the *Kiremt* season and harvested during the *Bega* season, while chickpea is planted in the last days of *Kiremt* and harvested in the *Bega* season. Thus, all six crops considered in this study are generally cultivated within a single cropping season.

Table 10: Decision variable and parameters.

Parameters	Description
TL	Total farmland for crop cultivation (ha)
TL_r	Minimum land area required for cultivation (ha)
Y_c	Average yield per unit area of crop c (qtl/ha)
N_c	Average net profit of crop c per hectare (ETB/ha)
I_c	Average investment per unit area of crop c (ETB/ha)
M_c	Average machine-hours required per unit area of crop c (hr/ha)
B_c	Average labor required per unit area of crop c (md/ha)
W_c	Average water requirement per unit area of crop c (m^3/ha)
E_c	Seed required per unit area of crop c (kg/ha)
F_{lc}	l^{th} type of fertilizer required per unit area for crop c (kg/ha)
P_{lc}	l^{th} type of pesticide required per unit area for crop c (L/ha)
H_{lc}	l^{th} type of herbicide required per unit area for crop c (L/ha)
TM	Total available machine-hours (hr)
TW	Total available water (m^3)
TE_c	Total available seed for crop c (kg)
TF_l	Total available l^{th} fertilizer (kg)
TP_l	Total available l^{th} pesticide (L)
TH_l	Total available l^{th} herbicide (L)
Decision variable	Description
X_c	Land area allocated to crop c (ha)

Note: md denotes man-days.

Objective functions
Based on their accessibility and regional importance, the following four objectives are considered in this study.
(i) Crop yield achievement
The estimated yield of a crop is equal to the product of the cultivable area of land and the average yield produced per unit area of land.
Thus, the maximization of the total yield of the considered six crops can be expressed as

$$\max \widetilde{f_1}(X) \approx \sum_{c=1}^{6} \widetilde{Y_c} X_c. \tag{27}$$

(ii) Net profit goal
The net profit of various crops, is the product of the net profit of each crop per unit area of land and its respective utilized land, which is described as

$$\max \widetilde{f_2}(X) \approx \sum_{c=1}^{6} \widetilde{N_c} X_c. \tag{28}$$

(iii) Cost of cultivation goal
To get the optimum production, farmers should invest a certain amount of money for land rent, fertilizers, seeds, herbicides, pesticides, rental machines, labor force, etc. Minimizing this working capital is another important objective of farmers and mathematically given by

$$\min \widetilde{f_3}(X) \approx \sum_{c=1}^{6} \widetilde{I_c} X_c. \tag{29}$$

(vi) Labor requirement
The labor objective is described as

$$\min \widetilde{f_4}(X) \approx \sum_{c=1}^{n} \widetilde{B_c} X_c. \tag{30}$$

Constraints
The above four objectives are subject to the following eight constraints.
(ii) Water requirement
Additional water through irrigation is required to meet the crop's evapotranspiration needs and optimize yield. The constraint for water supply can be described as

$$\sum_{c=1}^{6} \widetilde{W_c} X_c \precsim \widetilde{TW}. \tag{31}$$

For sustainable optimal crop yield and maximum profit, agricultural input resources should not be used at the expense of the environment (Li et al., 2020). Therefore, to reduce the adverse effects of fertilizers, pesticides, and herbicides on the environment, the optimum amounts of these inputs must be considered alongside the utilization of other favorable resources.
(iii) Dosage of fertilizer
To maintain and improve the productivity of the soil, different types of fertilizers have to be used optimally according to the characteristics of the crops, soil type and climate of the region. This constraint is expressed as

$$\sum_{c=1}^{6} \widetilde{F_{lc}} X_c \precsim \widetilde{TF_l},\ l = 1, 2. \tag{32}$$

(iv) Amount of Herbicide
DM requires a certain level of herbicides for several crops during growth stage for better yield taking the availability and environmental aspects into account. Mathematically described as

$$\sum_{c=1}^{6} \widetilde{H_{l_c}} X_c \precsim \widetilde{TH_l},\ l = 1, 2, ..., L. \tag{33}$$

(v) Dosage of pesticide
Pesticide is another essential input resource of agriculture to protect crops during growth stage from different pests and insects. This constraint is formulated as

$$\sum_{c=1}^{6} \widetilde{P_{lc}} X_c \precsim \widetilde{TP_l},\ l = 1, 2, ..., L. \tag{34}$$

(vi) Machine hours
Different types of machines are needed for various tasks of agriculture, such as tilling, plowing, sowing, cultivating, harvesting, threshing, etc. The sum of the machine hours allocated to each season should not exceed the machine hours required in a year. That means,

$$\sum_{c=1}^{6} \widetilde{M_c} X_c \precsim \widetilde{TM}. \tag{35}$$

(vii) Cultivable land availability
The sum of cultivable land for all crops must not exceed the total available land. Furthermore, the total cultivable land should be less than the entire arable land available in the study area. This is formulated as

$$\sum_{c=1}^{6} X_c \precsim \widetilde{TL}. \tag{36}$$

On the other hand, a minimum cultivable land area should be allocated to crop production to maintain a minimum level of agricultural output and profit while ensuring efficient utilization of available land resources. This constraint is mathematically expressed as

$$\sum_{c=1}^{6} X_c \succsim \widetilde{TL_r}. \tag{37}$$

(i) Seed requirement
Seed availability constraints were incorporated into the model to describe the limitations in accessing selected seed for each crop. The maximum seed availability for each crop was determined based on the recommended seed requirement per hectare (Table 7), the maximum feasible cultivable area for each crop, and crop suitability conditions under local agro-ecological settings (Semu et al., 2022). This constraint mathematically expressed as

$$\widetilde{E_c} X_c \precsim \widetilde{TE_c}, \quad c = 1, 2, ..., 6. \tag{38}$$

(viii) Non-negativity
In the modelling APP problem, all decision variables should be non-negative.

$$X_c \geq 0 \qquad c = 1, 2, ..., 6. \tag{39}$$

The stated objectives and constraints align with economic, environmental, and social goals, with aims to boost net benefits, use resources wisely to limit environmental harm, and increase local jobs (Li et al., 2020; Zhang & Georgescu, 2022).

In this study, the three variants of the problem are considered independently to address the interference of uncontrolled conditions and associated risks, and individual differences in the DM process. This assists farmers and managers from different perspectives by proposing various possible alternative management schemes.

In the optimistic assumption, a farmer considers using farming resources with varying degrees of acceptance and flexibility to accommodate other possible alternative operations, presuming that it offers certain benefits. Conversely, a pessimist DM tends to be skeptical about implementing all possible alternative farming tasks and partially considers those with a lower degree of acceptance. From a mixed perspective, the farmer's assumption lies between the optimistic and pessimistic viewpoints.
As a result, the solution of the APP problem varies according to each perspective (Kis et al., 2021), which leads to differences in the determination of the OCP. Each viewpoint has its advantages and disadvantages. Therefore, this study considers the optimistic, pessimistic, and mixed aspects of DM for efficient management of agricultural resources.

In this study, from $1,033ha$ land of the LSF, nearly $1,017ha$ of farmland is considered for crop cultivation, and $6ha$ to $7ha$ of land is supposed to be left permanently for forestation to maintain the ecological balance of the environment. In the existing system, $5ha$ of cropland is occupied by perennial crops. About $1.50ha$ of land of the LSF is permanently left over as a residential place for workers and roads. Moreover, $2.5ha$ of land is not suitable for farming and is currently used for animal grazing.

The land management of the LSF is a traditional approach based on the weather conditions. They interchangeably use plots of land for different crops, and there is no reasonable pattern of operation on cropland.

In this work, three scenarios of crop cultivation are designed based on the conventional cultivation pattern and suitability of the devised farming system in the study area.

The first scenario considers the case when chickpeas are planted on the land leftover in the first season of crop cultivation. Thus, in this scenario, chickpea faces the same land rent as other crops. This system is mainly adopted to fertilize unplowed land and to make use of fields left uncultivated in the first season due to factors like irregular rainfall, labor shortages, limited seed varieties, and lack of capital or fertilizers.

In the second scenario, farmers plant chickpeas on land right after harvesting maize. This common practice helps to use the fertile soil left by maize and saves farmers from the additional cost of renting extra land for smallholder farmers with limited fields to cultivate multiple crops. Based on the existing farming system in the study area, at least 33% of the land allocated to maize is subsequently used for chickpea cultivation after maize harvest.

In the third scenario, chickpeas are planted after harvesting maize, sorghum, and beans. Like the second scenario, the most important reason for farmers to use this cropping plan is to increase the yield from fertile land and minimize additional cultivation expenses. In order to effectively utilize the land for chickpea after harvesting maize, sorghum, and bean, the land allocated to chickpea should not be less than the combined land allocated to *teff* and pepper. Furthermore, the land allocated to bean should be at least equal to that allocated to sorghum. However, according to the farmers, this scenario is rarely practiced as land preparation for the succeeding crop after harvesting sorghum is a relatively challenging activity.

As chickpea is cultivated as a second crop after harvesting other crops within the same cropping season, the land rent for chickpea under the second and third scenarios decreases significantly by 65% relative to the annual land lease cost. Thus, to effectively utilize the available land in this production cycle, the upper limits of some constraints in the second and third scenarios are relaxed based on resource availability.

In the problem formulation of the three scenarios, the upper and lower limits of the constraints, the value of violations, and tolerances are mainly based on the availability of agricultural resources. For the second and third scenarios, the land constraint attributed to chickpeas is expressed in terms of the remaining crops.

The weights assigned to the objectives and constraints are estimated based on the preferences rated by managers of the LSF, farmers, and developmental agents of the study area.

Scenario 1: When chickpea is planted on the fallow land.

6 Results and Discussions

We denote the land areas allocated to crop c by x_c, where $c = 1, 2, ..., 6$, representing the land areas of maize, *teff*, sorghum, pepper, chickpea, and bean, respectively.

The functions $f_j(X)$, where $j = 1, 2, 3, 4$, denote the objectives of production, profit, expenditure, and manpower, respectively. The constraints are denoted by $g_i(X)$, where $i = 1, 2, ..., 8$, corresponding respectively to water, first-round fertilizer, second-round fertilizer, first-round herbicide, second-round herbicide, pesticide, machine hours, and the maximum available land area. In addition, minimum land area cultivation requirements are imposed to secure the minimum profit and yield, while seed constraints are included to account for limitations in seed availability.

Based on the available agricultural resources, the violation parameters of the constraints are assigned as $\ell_1 = 5235$, $\ell_2 = 935$, $\ell_3 = 940$, $\ell_4 = 96$, $\ell_5 = 88$, $\ell_6 = 78$, $\ell_7 = 104$, and $\ell_8 = 4$. The violation parameters for the seed constraints of maize, *teff*, sorghum, pepper, chickpea, and bean are assigned as 66, 40, 10, 10, 44, and 38, respectively, based on their availabilities.

The weights assigned to the production, profit, cost, and labor force objectives are 30%, 28%, 28%, and 14%, respectively. Utilizing all these values, the formulated problem is solved under optimistic, pessimistic, and mixed viewpoints by employing models (22), (23), and (24), respectively. The problem is defined using tolerance values expressed as multiples of λ, with violations evaluated under the three approaches. To explore different solutions, several values of $\lambda \in (0, 1)$, specifically $0.30, 0.40, 0.50, 0.60, 0.70,$ and 0.80, are considered in the solution process. Accordingly, the APP problem is solved for the three scenarios based on the objectives and constraints outlined in Section 5.3 and the data presented in Tables 5 to 8.

The mathematical expression of this problem has the following form:

$$
\begin{aligned}
&\max \widetilde{f}_1(X) = \widetilde{56}x_1 \oplus \widetilde{14.5}x_2 \oplus \widetilde{25}x_3 \oplus \widetilde{13.5}x_4 \oplus \widetilde{18}x_5 \oplus \widetilde{23.5}x_6,\\
&\max \widetilde{f}_2(X) = \widetilde{33125}x_1 \oplus \widetilde{13295}x_2 \oplus \widetilde{18154}x_3 \oplus \widetilde{192881}x_4 \oplus \widetilde{24342}x_5 \oplus \widetilde{16269}x_6,\\
&\min \widetilde{f}_3(X) = \widetilde{35625}x_1 \oplus \widetilde{3705}x_2 \oplus \widetilde{26845}x_3 \oplus \widetilde{41119}x_4 \oplus \widetilde{20659}x_5 \oplus \widetilde{25132}x_6,\\
&\min \widetilde{f}_4(X) = \widetilde{84}x_1 \oplus \widetilde{74}x_2 \oplus \widetilde{41}x_3 \oplus \widetilde{100}x_4 \oplus \widetilde{31}x_5 \oplus \widetilde{50}x_6\\
&\qquad\text{subject to}\\
&\widetilde{3610}x_1 \oplus \widetilde{2620}x_2 \oplus \widetilde{3245}x_3 \oplus \widetilde{4765}x_4 \oplus \widetilde{2540}x_5 \oplus \widetilde{2365}x_6 \precsim \widetilde{3410610},\\
&\widetilde{100}x_1 \oplus \widetilde{100}x_2 \oplus \widetilde{100}x_3 \oplus \widetilde{100}x_4 \oplus \widetilde{100}x_5 \oplus \widetilde{100}x_6 \precsim \widetilde{101700},\\
&\widetilde{125}x_1 \oplus \widetilde{100}x_2 \oplus \widetilde{100}x_3 \oplus \widetilde{200}x_4 \precsim \widetilde{108055},\\
&\widetilde{1}x_2 \oplus \widetilde{1.5}x_4 \oplus \widetilde{1}x_5 \oplus \widetilde{1}x_6 \precsim \widetilde{1145},\\
&\widetilde{1}x_1 \oplus \widetilde{0.5}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{0.75}x_6 \precsim \widetilde{1017},\\
&\widetilde{1}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{1.5}x_4 \oplus \widetilde{1}x_5 \oplus \widetilde{1}x_6 \precsim \widetilde{1100},\\
&\widetilde{2.67}x_1 \oplus \widetilde{2.67}x_2 \oplus \widetilde{2}x_3 \oplus \widetilde{2}x_4 \oplus \widetilde{2.67}x_5 \oplus \widetilde{2}x_6 \precsim \widetilde{2205},\\
&\widetilde{1}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{1}x_4 \oplus \widetilde{1}x_5 \oplus \widetilde{1}x_6 \precsim \widetilde{1017},\\
&\widetilde{1}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{1}x_4 \oplus \widetilde{1}x_5 \oplus \widetilde{1}x_6 \succsim \widetilde{1015},\\
&\widetilde{26}x_1 \precsim \widetilde{21611},\ \widetilde{30}x_2 \precsim \widetilde{15255},\ \widetilde{14.5}x_3 \precsim \widetilde{99156},\\
&\widetilde{15.5}x_4 \precsim \widetilde{6509},\ \widetilde{28}x_5 \precsim \widetilde{15662},\ \widetilde{20}x_6 \precsim \widetilde{12204},\\
&X = (x_1, x_2, x_3, x_4, x_5, x_6) \geq 0,
\end{aligned}
\tag{40}
$$

where,

$\widetilde{1145} = \langle 1140, 1145, 1146;\ 1139, 1145, 1147\rangle$,
$\widetilde{1017} = \langle 1016, 1017, 1018;\ 1015, 1017, 1019\rangle$,
$\widetilde{1015} = \langle 1014, 1015, 1016;\ 1013, 1015, 1017\rangle$,
$\widetilde{1100} = \langle 1099, 1101, 1107;\ 1098, 1101, 1108\rangle$,
$\widetilde{2205} = \langle 2200, 2205, 2207;\ 2197, 2205, 2208\rangle$,
$\widetilde{1} = \langle 0.95, 0.99, 1;\ 0.94, 0.99, 1.15\rangle$,
$\widetilde{3410610} = \langle 3410609, 3410610, 3410615;\ 3410607, 3410610, 3410617\rangle$,
$\widetilde{101700} = \langle 101697, 101699, 101705;\ 101695, 101699, 101707\rangle$,
$\widetilde{108055} = \langle 108052, 108057, 108058;\ 108050, 108057, 108060\rangle$,
$\widetilde{21611} = \langle 21605, 21610, 21611; 21604, 21610, 21612\rangle$,
$\widetilde{15255} = \langle 15250, 15255, 15256;\ 15249, 15255, 15257\rangle$,
$\widetilde{99156} = \langle 99151, 99156, 99157;\ 99150, 99156, 99158\rangle$,
$\widetilde{15662} = \langle 15657, 15662, 15663;\ 15656, 15662, 15664\rangle$,

$\widetilde{12204} = \langle 12199, 12204, 12205;\ 12198, 12204, 12206\rangle$,
$\widetilde{28138} = \langle 28132, 28138, 28140;\ 28130, 28138, 28142\rangle$,
$\widetilde{6509} = \langle 6504, 6509, 6510;\ 6503, 6509, 6511\rangle$.

The remaining values of the coefficients of the variables are presented in Tables 5 to 8.

The solutions of problem (40) obtained using the proposed optimistic approach for different $\lambda \in (0, 1)$ are presented in Table 11.

Table 11: Solutions of problem (40) under an optimistic perspective.

λ	X	$f_1(X)$	$f_2(X)$	$f_3(X)$	$f_4(X)$	Z
0.3	(153.44, 0, 314.97, 78.79, 152.82, 314.97)	27333.18	34842180	28234760	54150	0.2651
0.4	(162.93, 0, 305.66, 81.88, 158.86, 305.66)	27557.00	35579290	28340800	54630	0.2811
0.5	(171.96, 0, 296.48, 84.73, 165.36, 296.48)	27766.47	36269800	28436290	55064	0.2949
0.6	(180.58, 0, 287.40, 87.37, 172.26, 287.40)	27963.23	36919230	28522470	55464	0.3070
0.7	(187.95, 0, 278.65, 89.34, 180.41, 278.65)	28121.20	37441020	28579940	55761	0.3176
0.8	(191.60, 0, 270.90, 89.31, 192.28, 270.90)	28163.46	37578060	28551380	55751	0.3274

From the above solutions, Table 11, the better compromised solution is obtained when $\lambda = 0.8$. So, the compromised solution for the first scenario is $X = (191.60, 0, 270.90, 89.31, 192.28, 270.90)$ and its detail is depicted in Table 12.

Table 12: Compromise solution of problem (40).

$X=(x_1,x_2,x_3,x_4,x_5,x_6)$		$f_1(X)$	$f_2(X)$	$f_3(X)$	$f_4(X)$
(191.60, 0, 270.90, 89.31, 192.28, 270.90)		28163.46	37578060	28551380	55751
	μ	0.48	0.50	0.37	0.31
	ν	0.20	0.20	0.26	0.29
$X=(x_1,x_2,x_3,x_4,x_5,x_6)$		$g_1(X)$	$g_2(X)$	$g_3(X)$	$g_4(X)$
(191.60, 0, 270.90, 89.31, 192.28, 270.90)		3124448	101500	68902	597.15
$X=(x_1,x_2,x_3,x_4,x_5,x_6)$		$g_5(X)$	$g_6(X)$	$g_7(X)$	$g_8(X)$
(191.60, 0, 270.90, 89.31, 192.28, 270.90)		665.68	1059.65	2287	1015

Based on the results given in Table 12, the required input resources to cultivate the crops under the allocated cropland are presented in Table 13.

Table 13: Input resource requirements for allocated crops under scenario one.

Crop	Seed (kg)	F_{1c_n} (kg)	F_{2c_n} (kg)	H_{1c_n} (L)	H_{2c_n} (L)	P_{c_n} (L)
maize (c_1)	4789.95	15344	23949.79	-	153.44	153.44
sorghum (c_3)	4063.58	31497	27090.56	-	314.97	314.97
pepper (c_4)	1428.93	7879	17861.63	133.96	-	133.96
chickpea (c_5)	5383.90	15282	-	192.28	-	152.82
bean (c_6)	5418.11	31497	-	270.90	203.17	314.97

Scenario 2: Chickpea is planted after harvesting maize.
The mathematical expression of this problem has the following form:

$$
\begin{aligned}
&\max \widetilde{f_1}(X) = \widetilde{56}x_1 \oplus \widetilde{14.5}x_2 \oplus \widetilde{25}x_3 \oplus \widetilde{13.5}x_4 \oplus \widetilde{18}x_5 \oplus \widetilde{23.5}x_6,\\
&\max \widetilde{f_2}(X) = \widetilde{36100}x_1 \oplus \widetilde{13295}x_2 \oplus \widetilde{18154}x_3 \oplus \widetilde{192881}x_4 \oplus \widetilde{29865}x_5 \oplus \widetilde{16269}x_6,\\
&\min \widetilde{f_3}(X) = \widetilde{32650}x_1 \oplus \widetilde{3705}x_2 \oplus \widetilde{26845}x_3 \oplus \widetilde{41119}x_4 \oplus \widetilde{15135}x_5 \oplus \widetilde{25132}x_6,\\
&\min \widetilde{f_4}(X) = \widetilde{84}x_1 \oplus \widetilde{74}x_2 \oplus \widetilde{41}x_3 \oplus \widetilde{100}x_4 \oplus \widetilde{31}x_5 \oplus \widetilde{50}x_6\\
&\qquad\qquad\qquad\text{subject to}\\
&\widetilde{3610}x_1 \oplus \widetilde{2620}x_2 \oplus \widetilde{3245}x_3 \oplus \widetilde{4765}x_4 \oplus \widetilde{2540}x_5 \oplus \widetilde{2365}x_6 \precsim \widetilde{3667300},\\
&\widetilde{100}x_1 \oplus \widetilde{100}x_2 \oplus \widetilde{100}x_3 \oplus \widetilde{100}x_4 \oplus \widetilde{100}x_5 \oplus \widetilde{100}x_6 \precsim \widetilde{105900},\\
&\widetilde{125}x_1 \oplus \widetilde{100}x_2 \oplus \widetilde{100}x_3 \oplus \widetilde{200}x_4 \precsim \widetilde{127125},\\
&\widetilde{1}x_2 \oplus \widetilde{1.5}x_4 \oplus \widetilde{1}x_5 \oplus \widetilde{1}x_6 \precsim \widetilde{1145},\\
&\widetilde{1}x_1 \oplus \widetilde{0.5}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{0.75}x_6 \precsim \widetilde{1017},\\
&\widetilde{1}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{1.5}x_4 \oplus \widetilde{1}x_5 \oplus \widetilde{1}x_6 \precsim \widetilde{1525},\\
&\widetilde{2.167}x_1 \oplus \widetilde{2.167}x_2 \oplus \widetilde{2}x_3 \oplus \widetilde{2}x_4 \oplus \widetilde{2.167}x_5 \oplus \widetilde{2}x_6 \precsim \widetilde{2715},\\
&\widetilde{1}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{1}x_4 \oplus \widetilde{1}x_6 \precsim \widetilde{1017},\\
&\widetilde{1}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{1}x_4 \oplus \widetilde{1}x_6 \succsim \widetilde{1015},\\
&\widetilde{1}x_1 \succsim \widetilde{1}x_5,\ \widetilde{1}x_5 \succsim \widetilde{0.33}x_1,\\
&\widetilde{26}x_1 \precsim \widetilde{21611},\ \widetilde{30}x_2 \precsim \widetilde{15255},\ \widetilde{14.5}x_3 \precsim \widetilde{99156},\\
&\widetilde{15.5}x_4 \precsim \widetilde{6509},\ \widetilde{28}x_5 \precsim \widetilde{15662},\ \widetilde{20}x_6 \precsim \widetilde{12204},\\
&X=(x_1,x_2,x_3,x_4,x_5,x_6) \geq 0,
\end{aligned}
\tag{41}
$$

where

$$
\begin{gathered}
\widetilde{3667300} = \langle 3667293, 3667304, 3667307; 3667290, 3667304, 3667310\rangle,\\
\widetilde{2715} = \langle 2712, 2715, 2718; 2710, 2715, 2720\rangle, \widetilde{0.33} = \langle 0.31, 0.33, 0.35; 0.30, 0.33, 0.36\rangle,\\
\widetilde{105900} = \langle 105895, 105900, 105905; 105890, 105900, 105910\rangle,\\
\widetilde{127125} = \langle 127122, 127125, 127128; 127120, 127125, 127130\rangle,\\
\widetilde{29490} = \langle 29491, 29492, 29498; 29488, 29492, 29499\rangle,\\
\widetilde{36100} = \langle 36096, 36098, 36104; 36095, 36098, 36105\rangle,\\
\widetilde{15135} = \langle 15132, 15134, 15140; 15130, 15134, 15142\rangle,\\
\widetilde{29865} = \langle 29862, 29865, 29868; 29860, 29865, 29870\rangle,\\
\widetilde{32650} = \langle 32648, 32650, 32655; 32645, 32650, 32660\rangle,\\
\widetilde{1525} = \langle 1522, 1525, 1528; 1520, 1525, 1530\rangle.
\end{gathered}
$$

By solving problem (41) using the proposed method for the optimistic variants of the problem, a better compromise solution is obtained for $\lambda = 0.8$. The resulting compromised optimal solution for the second scenario is $X = (156.00, 0.00, 661.58, 122.85, 51.48, 74.57)$, with details presented in Table 14.

Table 14: Compromise solution of problem (41).

$X = (x_1, x_2, x_3, x_4, x_5, x_6)$		$f_1(X)$	$f_2(X)$	$f_3(X)$	$f_4(X)$
(156.00, 0.00, 661.58, 122.85, 51.48, 74.57)		29358.32	44088180	30558420	57384
	μ	0.92	0.30	0.36	0.54
	ν	0.03	0.30	0.26	0.18
$X = (x_1, x_2, x_3, x_4, x_5, x_6)$		$g_1(X)$	$g_2(X)$	$g_3(X)$	$g_4(X)$
(156.00, 0.00, 661.58, 122.85, 51.48, 74.57)		3601808	106648	110228	310.32
$X = (x_1, x_2, x_3, x_4, x_5, x_6)$		$g_5(X)$	$g_6(X)$	$g_7(X)$	$g_8(X)$
(156.00, 0.00, 661.58, 122.85, 51.48, 74.57)		873.51	1127.90	2271.97	1015

The required input resources to cultivate the crops under the allocated cropland are presented in Table 15.

Table 15: Input resource requirements for allocated crops under scenario two.

Crop	Seed (kg)	F_{1c_n} (kg)	F_{2c_n} (kg)	H_{1c_n} (L)	H_{2c_n} (L)	P_{c_n} (L)
maize (c_1)	3900.00	15600	19500	-	156.00	156.00
sorghum (c_3)	9923.75	66158	66158	-	661.58	661.58
pepper (c_4)	1965.58	12285	24570	184.27	-	184.27
chickpea (c_5)	1441.44	5148	-	51.48	-	51.48
bean (c_6)	1491.36	7457	-	74.57	55.93	74.57

Scenario 3: Chickpea is planted after harvesting maize, sorghum and beans.
The mathematical expression of this scenario has the following form:

$$
\begin{aligned}
&\max \widetilde{f}_1(X) = \widetilde{56}x_1 \oplus \widetilde{14.5}x_2 \oplus \widetilde{25}x_3 \oplus \widetilde{13.5}x_4 \oplus \widetilde{18}x_5 \oplus \widetilde{23.5}x_6,\\
&\max \widetilde{f}_2(X) = \widetilde{36100}x_1 \oplus \widetilde{13295}x_2 \oplus \widetilde{21130}x_3 \oplus \widetilde{192881}x_4 \oplus \widetilde{29865}x_5 \oplus \widetilde{19245}x_6,\\
&\min \widetilde{f}_3(X) = \widetilde{32650}x_1 \oplus \widetilde{3705}x_2 \oplus \widetilde{23870}x_3 \oplus \widetilde{41119}x_4 \oplus \widetilde{15135}x_5 \oplus \widetilde{22155}x_6,\\
&\min \widetilde{f}_4(X) = \widetilde{84}x_1 \oplus \widetilde{74}x_2 \oplus \widetilde{41}x_3 \oplus \widetilde{100}x_4 \oplus \widetilde{31}x_5 \oplus \widetilde{50}x_6\\
&\qquad\qquad \text{subject to}\\
&\widetilde{3610}x_1 \oplus \widetilde{2620}x_2 \oplus \widetilde{3245}x_3 \oplus \widetilde{4765}x_4 \oplus \widetilde{2540}x_5 \oplus \widetilde{2365}x_6 \precsim \widetilde{3667300},\\
&\widetilde{100}x_1 \oplus \widetilde{100}x_2 \oplus \widetilde{100}x_3 \oplus \widetilde{100}x_4 \oplus \widetilde{100}x_5 \oplus \widetilde{100}x_6 \precsim \widetilde{105900},\\
&\widetilde{125}x_1 \oplus \widetilde{100}x_2 \oplus \widetilde{100}x_3 \oplus \widetilde{200}x_4 \precsim \widetilde{127125},\\
&\widetilde{1}x_2 \oplus \widetilde{1.5}x_4 \oplus \widetilde{1}x_5 \oplus \widetilde{1}x_6 \precsim \widetilde{1145},\\
&\widetilde{1}x_1 \oplus \widetilde{0.5}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{0.75}x_6 \precsim \widetilde{1017},\\
&\widetilde{1}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{1.5}x_4 \oplus \widetilde{1}x_5 \oplus \widetilde{1}x_6 \precsim \widetilde{1525},\\
&\widetilde{2.167}x_1 \oplus \widetilde{2.167}x_2 \oplus \widetilde{2}x_3 \oplus \widetilde{2}x_4 \oplus \widetilde{2.167}x_5 \oplus \widetilde{2}x_6 \precsim \widetilde{2715},\\
&\widetilde{1}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{1}x_4 \oplus \widetilde{1}x_6 \precsim 1017,\\
&\widetilde{1}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{1}x_3 \oplus \widetilde{1}x_4 \oplus \widetilde{1}x_6 \succsim 1015,\\
&\widetilde{1}x_5 \precsim \widetilde{1}x_1 \oplus \widetilde{1}x_3 \oplus \widetilde{1}x_6,\ \widetilde{1}x_5 \succsim \widetilde{1}x_2 \oplus \widetilde{1}x_4,\ \widetilde{1}x_6 \succsim \widetilde{1}x_3.\\
&\widetilde{26}x_1 \precsim \widetilde{21611},\ \widetilde{30}x_2 \precsim \widetilde{15255},\ \widetilde{14.5}x_3 \precsim \widetilde{99156},\\
&\widetilde{15.5}x_4 \precsim \widetilde{6509},\ \widetilde{28}x_5 \precsim \widetilde{15662},\ \widetilde{20}x_6 \precsim \widetilde{12204},\\
&X = (x_1, x_2, x_3, x_4, x_5, x_6) \geq 0,
\end{aligned}
\tag{42}
$$

where
$$
\begin{aligned}
\widetilde{21130} &= \langle 21126, 21130, 21134; 21124, 21130, 21136\rangle,\\
\widetilde{23870} &= \langle 23864, 23870, 23876; 23862, 23870, 23878\rangle,\\
\widetilde{19245} &= \langle 19240, 19245, 19247; 19237, 19245, 19248\rangle,\\
\widetilde{22155} &= \langle 22153, 22155, 22160; 22152, 22155, 22163\rangle.
\end{aligned}
$$

By solving problem (42) using the proposed method for the optimistic variants of the problem, a compromise solution is obtained for $\lambda = 0.8$. The resulting compromised optimal solution for the third scenario is $X = (409.19, 0.00, 277.16, 51.48, 51.48, 277.16)$, with details presented in Table 16.

Table 16: Compromise solution of problem (42).

$X = (x_1, x_2, x_3, x_4, x_5, x_6)$		$f_1(X)$	$f_2(X)$	$f_3(X)$	$f_4(X)$
(409.19, 0.00, 277.16, 51.48, 51.48, 277.16)		37405.40	37428600	29013310	66522
	μ	0.41	0.72	0.33	0.38
	ν	0.24	0.10	0.28	0.26
$X = (x_1, x_2, x_3, x_4, x_5, x_6)$		$g_1(X)$	$g_2(X)$	$g_3(X)$	$g_4(X)$
(409.19, 0.00, 277.16, 51.48, 51.48, 277.16)		3406694	106648	89162	405.86
$X = (x_1, x_2, x_3, x_4, x_5, x_6)$		$g_5(X)$	$g_6(X)$	$g_7(X)$	$g_8(X)$
(409.19, 0.00, 277.16, 51.48, 51.48, 277.16)		894.23	1092.22	2441.61	1015

The required input resources to cultivate the crops under the allocated cropland are presented in Table 17.

Table 17: Input resource requirements for allocated crops under scenario three.

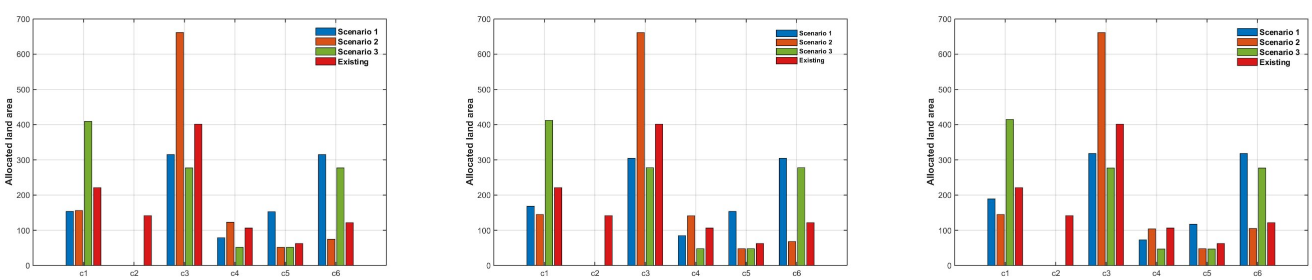
Crop	Seed (kg)	F_{1c_n} (kg)	F_{2c_n} (kg)	H_{1c_n} (L)	H_{2c_n} (L)	P_{c_n} (L)
maize (c_1)	10229.87	40919	51149.35	-	409.19	409.19
sorghum (c_3)	4157.44	27716	27716.26	-	277.16	277.16
pepper (c_4)	823.68	5148	10296.00	77.22	-	77.22
chickpea (c_5)	1441.44	5148	-	51.48	-	51.48
bean (c_6)	5543.25	27716	-	277.16	207.87	277.16

Similarly, the considered problem under the three scenarios can also be solved for pessimistic and mixed DMs. The compromise solutions for each scenario under pessimistic and mixed viewpoints are presented in the upper and lower parts of Table 18, respectively.

Table 18: Solutions under pessimistic and mixed perspectives.

Scenario	λ	X	Z
1	0.40	$(168.27, 0.00, 304.30, 84.61, 153.51, 304.30)$	0.4132
2	0.40	$(144.67, 0.00, 661.32, 141.05, 47.74, 67.97)$	0.4626
3	0.40	$(412.14, 0.00, 277.56, 47.74, 47.74, 277.56)$	0.4495
1	0.30	$(189.12, 0.00, 317.99, 72.76, 117.14, 317.99)$	0.4067
2	0.40	$(144.67, 0.00, 661.32, 104.06, 47.74, 104.95)$	0.4352
3	0.30	$(414.56, 0.00, 276.82, 46.80, 46.80, 276.81)$	0.4491

Based on the obtained results, the arable land allocated to the six crops under the three approaches regarding the conventional pattern and the three scenarios is presented in Figure 5.



(a) (b) (c)

Figure 5: Allocated land area to the six crops under the optimistic (a), pessimistic (b) and mixed (c) perspectives relative to the existing pattern.

Let X_1, X_2, and X_3 denote the compromised solutions for the first, second, and third scenarios, respectively, under pessimistic and mixed views (Table 18). The objective values, along with the membership and non-membership degrees of the objectives for the pessimistic and mixed variants of the problem, are presented in Table 19.

Table 19: Goal achievement under pessimistic and mixed perspectives.

	Pessimistic perspective				Mixed perspective			
	$f_1(X_1)$	$f_2(X_1)$	$f_3(X_1)$	$f_4(X_1)$	$f_1(X_1)$	$f_2(X_1)$	$f_3(X_1)$	$f_4(X_1)$
$f(X_1)$	27724.77	36105370	28462070	55064	28719.63	34095870	28677430	55719
μ	0.44	0.45	0.38	0.34	0.54	0.40	0.35	0.31
ν	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
	$f_1(X_2)$	$f_2(X_2)$	$f_3(X_2)$	$f_4(X_2)$	$f_1(X_2)$	$f_2(X_2)$	$f_3(X_2)$	$f_4(X_2)$
$f(X_2)$	28745.80	46965540	30707130	57780	29115.66	40433450	30115760	55931
μ	0.78	0.34	0.34	0.52	0.86	0.25	0.41	0.58
ν	0.00	0.00	0.00	0.00	0.00	0.21	0.06	0.00
	$f_1(X_3)$	$f_2(X_3)$	$f_3(X_3)$	$f_4(X_3)$	$f_1(X_3)$	$f_2(X_3)$	$f_3(X_3)$	$f_4(X_3)$
$f(X_3)$	37470.45	36717860	28917350	66314	37538.86	36566940	28909570	66329
μ	0.42	0.68	0.35	0.38	0.42	0.67	0.35	0.38
ν	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Based on the obtained solutions using the pessimistic and mixed approaches (Table 18), the agricultural resource consumptions for the three scenarios under each perspective, in order, are presented in Tables 20 and 21.

Table 20: Resource consumption under the pessimistic viewpoint.

$g_1(X_1)$	$g_2(X_1)$	$g_3(X_1)$	$g_4(X_1)$	$g_5(X_1)$	$g_6(X_1)$	$g_7(X_1)$	$g_8(X_1)$
3106967	101500	68386.55	584.73	700.80	1057.30	2245.60	1015.00
$g_1(X_2)$	$g_2(X_2)$	$g_3(X_2)$	$g_4(X_2)$	$g_5(X_2)$	$g_6(X_2)$	$g_7(X_2)$	$g_8(X_2)$
3621577	106274	112424.80	327.28	856.96	1133.26	2254.39	1015
$g_1(X_3)$	$g_2(X_3)$	$g_3(X_3)$	$g_4(X_3)$	$g_5(X_3)$	$g_6(X_3)$	$g_7(X_3)$	$g_8(X_3)$
3392248	106274	88821.33	396.91	897.87	1086.61	2433.60	1015

Table 21: Resource consumption under the mixed viewpoint.

$g_1(X_1)$	$g_2(X_1)$	$g_3(X_1)$	$g_4(X_1)$	$g_5(X_1)$	$g_6(X_1)$	$g_7(X_1)$	$g_8(X_1)$
3110209	101500	69990.82	544.27	745.60	1051.38	2235.20	1015
$g_1(X_2)$	$g_2(X_2)$	$g_3(X_2)$	$g_4(X_2)$	$g_5(X_2)$	$g_6(X_2)$	$g_7(X_2)$	$g_8(X_2)$
3533070	106274	105027.60	308.79	884.70	1114.77	2254.39	1015
$g_1(X_3)$	$g_2(X_3)$	$g_3(X_3)$	$g_4(X_3)$	$g_5(X_3)$	$g_6(X_3)$	$g_7(X_3)$	$g_8(X_3)$
3389983	106180.50	88862.42	393.83	898.99	1085.21	2432.72	1015

The OCP obtained using the proposed optimization method was compared with existing patterns. Since the second scenario represents a commonly practiced farming system in the district, a comparison was made between the existing cropping pattern and the optimized plan under the optimistic approach for this scenario.

As shown in Table 9, the area of land allocated to maize, *teff*, chickpea and bean in the existing situation was $221.03ha$, $141.41ha$, $62.29ha$, and 121.57, respectively, while in the proposed pattern, as presented in Table 14, the land area allocated to these crops respectively decreased to $156ha$, $0.00ha$, $51.45ha$, and 74.57. The area of land allocated to sorghum and pepper in the existing situation was $401.33ha$ and $106.42ha$, respectively, while in the proposed pattern, the land area allocated to these crops respectively increased to $661.58ha$ and 122.85.

In the conventional farming pattern, $991.76ha$ (96.00%) of $1,033ha$ of land is allocated to five crops in the first round of farming, and $62.29ha$ (28.18%) of $221.03ha$ of land is allocated to chickpea in the second round farming. In the proposed cropping pattern, $1,015ha$ (98.26%) of $1,033ha$ of land is allocated to four crops in the first season of farming and $51.48ha$ (33%) of $156ha$ of land is allocated to chickpea in the second round of farming.

The results of the study showed that, including in the remaining scenarios and DM approaches, *teff* should not be included in the farming patterns in favour of increasing the land area for other crops to attain better results regarding all objectives and constraints.

From the allocated arable land in the existing farm patterns, a total of $29,470.15qtl$ of yield was obtained under the existing cropping pattern, but this is slightly reduced to $29,358.32qtl$ in the proposed approach. If we consider the remaining objectives, the total gain was $41,510,180ETB$ in the existing pattern, but it can be increased to $44,088,180ETB$ applying the proposed farming pattern. On the other hand, the cost of cultivation and labor force were $31,413,470ETB$ and $64,160md$, respectively, in the existing farm plan, and these can be minimized to $30,558,420ETB$ and $57,384md$, respectively, by employing the suggested farming pattern.

Employing the proposed cropping pattern, the objectives are achieved with higher degrees of membership and lower degrees of non-membership compared to the existing cropping plan, except for the maximization of the yield target. However, in the existing situation, the production objective is accomplished at the cost of agrarian assets and the remaining goals.

There are notable differences between the conventional farming system and the suggested farming plan in the usage of manpower, fertilizers, and agricultural machines; whereas there are slight differences in the usage of herbicides and pesticides. If we consider the fertilizer constraint, $208,591.20kg$ of fertilizer was required to cultivate $1,054ha$ of farmland in the existing farming pattern, while the suggested farming plan requires $216,876kg$ of fertilizer to cultivate $1,066.48ha$ of farmland within the two production cycles. To cultivate the allocated crops on the respective areas of land, $2,392hrs$ (machine hrs.) were required using the existing plan, whereas the suggested plan require $2,272hrs$.

Comparing the obtained results of optimistic approach under the first, second, and third scenarios, Tables 12, 14, and 16, respectively, the following assessments have been made.
A comparatively wide area of land is allocated to sorghum and maize, under

the second and third scenarios, respectively, while pepper and bean are allocated in a wide area of farmland under the second and third scenarios, respectively. In contrast, Chickpeas share the largest area of land under the first scenario. Considering the resource constraints, water and fertilizer consumption can be minimized by applying the first farming scenario, while herbicide and pesticide consumption can be reduced by applying the second scenario. Whereas seed utilization can be minimized in the second scenario. In the achievement of the considered objectives, enhanced overall production and minimum expenditure are attained under the third scenario. While the total gain is significantly improved under the second scenario and the number of manpower is sufficiently minimized by applying the first farming scenario.

Based on the results of the three variants of the problem, as presented in Tables 12, 14, 16, 18, 19, 20, and 21 there is significant variation in the land allocation to the six crops under each scenario. This proves the solution of APP problem is contingent upon the DM's perspective. For example, in the first scenario, the land allocated to maize under the optimistic approach is reduced by $23.33ha$ and $2.48ha$, respectively, in the pessimistic and mixed approaches. Whereas the cropland allocated to bean in the mixed approach, respectively, decreased by $13.69ha$ and $47.09ha$ in the pessimistic and optimistic approaches.

The total cost of cultivation and manpower goals are better minimized under a pessimistic approach, while total crop production and profit are better achieved under the mixed and optimistic approaches, respectively.In the second scenario, a relatively equal large land area is allocated to sorghum under the three approaches, whereas for beans, a wide area is allocated under the mixed approach, but this is reduced by 30.38ha and 36.98ha under the optimistic and pessimistic approaches, respectively. The farmland allocated to maize and chickpea remains the same under the pessimistic and mixed approaches and differs by 11.33ha for maize and 3.74ha for chickpea under the optimistic approach. In this scenario, the yield maximization target is improved under the optimistic approach, while the profit maximization target is better achieved under the pessimistic approach. In contrast, the cost and manpower minimization targets are enhanced under the mixed approach.

In the third scenario, sorghum and beans share almost equal land areas under the three approaches. Moreover, a wide area of land is allocated to maize, with slight variations under the three approaches. The production and cost of cultivation goals are improved under the mixed approach, while the profit target is significantly maximized under the optimistic approach.

Water and fertilizer are fairly utilized under the pessimistic approach for the first scenario, whereas herbicide and pesticide consumption are minimized under the mixed approach.In the second scenario, water and herbicide usage are reduced under the mixed approach, while pesticide consumption is minimized under the pessimistic approach. In the third scenario, water consumption is significantly reduced under the pessimistic approach, while fertilizer and herbicide utilization are minimized under the pessimistic and mixed approaches, respectively.

7 Conclusion and Recommendations

In this paper, the APP problem is addressed using the IFMOO method from different perspectives, considering three common cropping scenarios in the study area. The resulting farming plans offer practical solutions aimed at reducing the overall vulnerability of six main crops in Abeshge district to various agricultural challenges.

The comparison assessment made between the previous farming system and the proposed cropping patterns verified that the proposed farming patterns have several advantages for better achievement of the stated objectives and efficient utilization of agricultural resources.

The existing farming system in the district relies heavily on capital and labor, often causing harm to the environment. If this continues, the area will face soil degradation unless resources are managed properly. All stakeholders need to take action, especially in reducing fertilizer use. For example, using manure and compost can help protect soil fertility, and practices like crop rotation can lower the need for herbicides.

The study has numerous benefits in assisting the managers of LSF and farmers of the district for optimal management of agricultural resources. It also indicates the advantages of OCP to overcome the potential disaster of crops due to climate change and soil infertility. Furthermore, the study can be used to predict promising cropping plans from a long-term perspective as well.

A proficient IFMOO model is proposed to address uncertainties and associated risks of agriculture, aiming to achieve sustainable crop production goals. Incorporating the risk management analysis model into the IFMOO model can increase the efficiency and applicability of the proposed approach to APP problems. Moreover, higher-order extensions of IFO techniques, such as hesitant IFO (Teferi et al., 2025), are also helpful in capturing the hesitation among DMs.

Data availability

The detailed experimental data used to support the findings of this study are included in the supplementary information file(s).

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Conflict of Interest

The author declares that he has no competing financial interests or personal relationships that could have influenced the work reported in this paper.

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